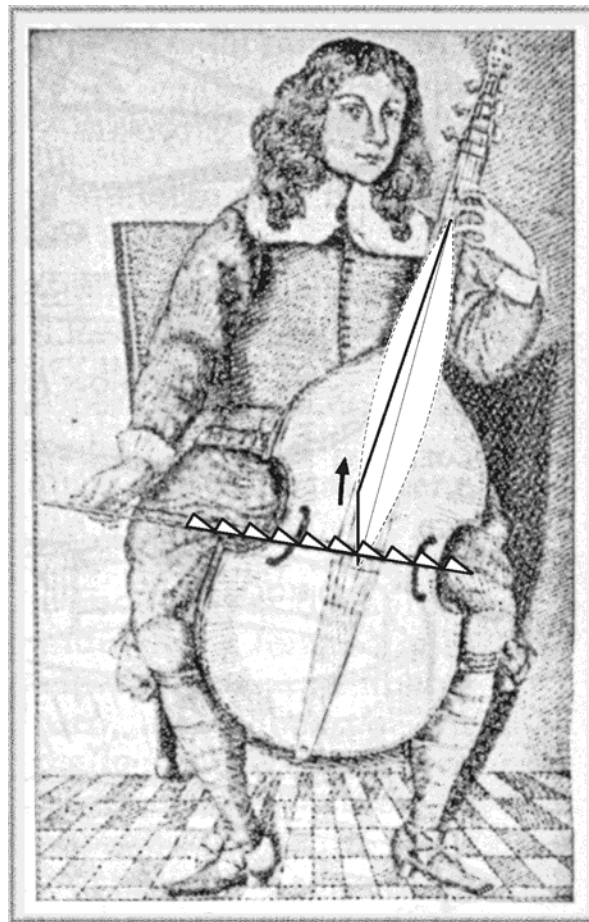




KUNGL
TEKNISKA
HÖGSKOLAN

The Bowed String

*On the Development of Helmholtz Motion
and
On the Creation of Anomalous Low Frequencies*



Knut Guettler

Doctorate thesis

Royal Institute of Technology - Speech, Music and Hearing
Stockholm 2002



KUNGL
TEKNISKA
HÖGSKOLAN

Royal Institute of Technology
Speech, Music and Hearing

The Bowed String
On the Development of Helmholtz Motion
and
On the Creation of Anomalous Low Frequencies

Knut Guettler

TRITA-TMH 02:3
ISSN 1100-5787
ISBN 91-7283-279-7

Doctorate thesis
April 5, 2002
KTH, Stockholm Sweden

Abstract

Of the many waveforms the bowed string can assume, the so-called “Helmholtz motion” (Helmholtz 1862) gives the fullest sound in terms of power and overtone richness. The development of this steady-state oscillation pattern can take many different paths, most of which would include noise caused by stick-slip irregularities of the bow-string contact. Of the five papers included in the thesis, the first one shows, not surprisingly, that tone onsets are considered superior when the attack noise has a very limited duration. It was found, however, that in this judgment the *character* of the noise plays an important part, as the listener’s tolerance of noise in terms of duration is almost twice as great for “slipping noise” as for “creaks” or “raucousness” during the tone onsets. The three following papers contain analyses focusing on how irregular slip-stick triggering may be avoided, as is quite often the case in practical playing by professionals. The fifth paper describes the triggering mechanism of a peculiar tone production referred to as “Anomalous Low Frequencies” (ALF). If properly skilled, a player can achieve pitches below the normal range of the instrument. This phenomenon is related to triggering waves taking “an extra turn” on the string before causing the string’s release from the bow-hair grip. Since transverse and torsional propagation speeds are both involved, two different sets of “sub-ranged” notes can be produced this way. In the four last papers wave patterns are analysed and explained through the use of computer simulations.

Key words:

Bowed string, violin, musical acoustics, musical transient, anomalous low frequencies, Helmholtz motion.

Contents

Preface	2
The Helmholtz motion	4
A brief description of the FIDDLE simulation program	4
♦ General description ♦ Friction models ♦ Special features	
♦ The time-dependent input variables ♦ Built-in tools for analysis ♦ “Potential friction force”	
Errata	8
Introduction and comments to Paper I	
Acceptance limits for the duration of pre-Helmholtz transients in bowed string attacks...	9
K. Guettler and Askenfelt, A., (1997) <i>J. Acoust. Soc. Am.</i> 101 (5), 2903-2913.	
♦ Introduction ♦ Acceptance of noise in bowed “neutral” attacks	
♦ Character of noise varying with the musical context ♦ Background for the study	
Introduction and comments to Paper II	
The bowed string computer simulated – some characteristic features of the attack	12
K. Guettler (1992) <i>Catgut Acoust. Soc. J.</i> Vol. 2, No. 2 (Series II), 22-26.	
♦ Introduction ♦ The resistance-reactance reflection function ♦ Bow compliance	
♦ Moving into steady-state oscillations	
Introduction and comments to Paper III	
On the kinematics of spiccato and ricochet bowing	15
K. Guettler and Askenfelt, A. (1998) <i>Catgut Acoust. Soc. J.</i> Vol. 3, No. 6 (Series II), 9-15.	
♦ Introduction ♦ The character of spiccato and ricochet	
♦ The trajectories of the two bow movements ♦ The four phases of spiccato	
♦ The kinematical difference between ricochet and spiccato	
♦ The bounce rate ♦ Adjustment of the bounce rate ♦ The point of percussion	
♦ Preventing the bow from further bouncing	
Introduction and comments to Paper IV	
On the creation of the Helmholtz movement in the bowed string	19
K. Guettler (2001) Revised version accepted for publication in <i>Acustica-acta acustica</i> , April 2002.	
♦ Introduction ♦ Applying Schelleng’s method for analysis of the transient	
♦ The increasing acceptance of bow speed during the transient	
♦ The problem of including torsion in the equations—“torsionally related ringing”	
Introduction and comments to Paper V	
Wave analysis of a string bowed to anomalous low frequencies	22
K. Guettler (1994) <i>Catgut Acoust. Soc. J.</i> Vol. 2, No. 6 (Series II), 8-14.	
♦ Introduction ♦ The “echoes” of a string flyback ♦ Transverse and torsional triggering	
♦ Spectra of the ALF tones	
Additional information on Papers I and III	24
How it started—Acknowledgments	25
References	26
Papers I-V	

Preface

In most musical contexts, the “classical” string player is aiming at a quick development of the Helmholtz motion during the tone onset of the bowed string. As shall be shown, considerable bowing precision is required in order to generate a fast tone buildup while avoiding onset noise, in terms of non-periodic patterns, or patterns lacking energy in sets of partials. Typically, the bowed-string transient falls into one of the following three categories:

- *Multiple slips*, where more than one slipping interval occur during each nominal fundamental period.
- *Periodic stick-slip from the very beginning*, giving periods equal or close to the period length of the Helmholtz motion (in this thesis referred to as “perfect attack”);
- *Prolonged irregular periods* (“creaky” or “raucous” sounds, usually with no clearly definable pitch).

In addition to these three, which are all frequently occurring during practical playing, a fourth category of some musical virtue exists:

- *Prolonged periods of identical length*, each period exhibiting only one slip, triggered by either transverse or torsional waves (Anomalous Low Frequencies).

The five papers of this thesis do all deal with one or more aspects of the four categories above:

Paper I (*Acceptance limits for the duration of pre-Helmholtz transients in bowed string attacks*) shows that string players are quite sensitive to the noises produced when the slipping intervals deviate substantially from the nominal fundamental period during the onset transient. For a violin G-string, based on a listening test, tolerance limits are suggested with respect to the duration of multiple slips and prolonged irregular periods, respectively.

Paper II (*The bowed string computer simulated – some characteristic features of the attack*) presents some parameters that are likely to influence the bowed-string transients. Special focus is on the force peak that occurs in the static-friction interval after the first slip. This peak is probably the most protruding obstacle in the creation of a regular slip-stick pattern. While the range of bow velocities acceptable for a given bow force is initially quite restricted, the velocity can be chosen much more freely as the transient decays.

In Paper III (*On the kinematics of spiccato and ricochet bowing*) the build-up and decay of spiccato and ricochet tones are analysed. These “off-string” bowing techniques, which also put the bow’s quality to test, require a high degree of skill to be properly performed. The study shows, however, that “perfect attacks” (i.e., starting transients with regular periodic triggering all the way) can be produced also when the bow is thrown onto the string, as in spiccato and ricochet.

Paper IV (*On the creation of the Helmholtz movement in the bowed string*) is a thorough examination of onset transients that quickly lead to Helmholtz motion. Based on analysis of the waves returning to the bow, certain bowing-parameter requirements can be defined for simple bowed-string models. Patterns similar to those outlined by these equations are, however, recognisable also in more complex models.

Paper V (*Wave analysis of a string bowed to anomalous low frequencies*) uses similar wave-analysis techniques to explain the peculiar phenomenon of Anomalous Low Frequencies (ALF). Here the string is forced to oscillate at frequencies substantially lower than its usual transverse fundamental mode. It was found that these frequencies might be divided into two groups, triggered by transverse and torsional waves, respectively.

With the exception of Paper I, computer modelling of the bowed string has been utilised for analyses in all the works above. A brief presentation of the computer program is included in this thesis.

The main body of research papers on the bowed string has focused on the string in steady-state oscillation. The few reports dealing with bowed starting transients have typically been describing them as “long lasting” without further exhaustive analyses. Backhaus (1932) compares the duration of a violin transient—the five lowest harmonics of the tone A₄ (435 Hz)—to the transient of a trumpet’s F₄ (340 Hz). While for the trumpet all five harmonics reach full amplitude within 40 milliseconds (most of them much earlier), the violin uses more than 100 ms to do the same. However, unpublished measurements of energy radiated from a violin (performed by Lars Henrik Morset at the Norwegian University of Science and Technology) suggest a rise time in the order of 16 - 18 ms for the violin body when excited directly at the bridge. For frequency bands above the main air and wood resonances, the response might be even faster. Melka (1970) found that regardless of pitch, in violin pizzicato no tone build-up lasted longer than 4 ms. This points to the energy build-up in the string as the limiting factor, or to the string’s delicate role in wave building when bowed, which again is largely a function of the player’s intention and skill. In the Backhaus plot, the first harmonic builds up considerably more slowly than the lowest four partials above. That indicates a triggering not regular from the onset. With regular triggering the first harmonic would develop together with the others.

In the case of Anomalous Low Frequencies, transients are very short, because wave patterns do not build up in the same way as for the Helmholtz motion. The forced oscillations of ALF never form a fixed wave pattern travelling along the string. Within each prolonged period, the string signal is required to be significantly modified one or more times in order to provide the delayed triggering necessary for ALF to exist.

The present works have to a large degree been taking the player’s perspective: “What can be done to create the wanted sound?” “Which are the reasons for attacks to fail?”—Questions familiar to all instrumentalists. However, unlike performers of most instruments, the string player is continuously influencing the transient during its development. The development of a stable tone is literally led by the hand all the way through. Dependent on frequency and bowing position, the string and the responding instrument are merely defining a frame, within which the player must try to manoeuvre. While this frame is system specific, the resulting transient is not. The papers of this thesis have for the most part been trying to describe the nature of such frames. What might eventually come out of the string still lies in the hands of the player.

Knut Guettler

Oslo, April 6. 2002.

The Helmholtz motion

Description of the Helmholtz motion (Helmholtz 1862) can be found in any textbook on the acoustics of musical instruments. A brief summary is given here:

When a bowed string oscillates in steady state, the string takes the shape of two (nearly) straight lines connected by a kink or corner that rotates in a parabolic path, see Fig. T1. The contact between the bow and string switches between two conditions: sticking and slipping. During the interval when the kink is travelling between the bow and the bridge (“slip”), the string moves in a direction opposite to that of the bow. The velocity is then $(1 - \beta)/\beta$ times higher than the (steady) bow velocity supplied by the player. β denotes the ratio between the bow’s distance from the bridge, and the total string length. During the remaining part of the cycle (“stick”), the string is stuck to the bow hair and consequently takes the speed of the bow. The sticking and the slipping intervals thus last $(\beta - 1)/f_0$ and β/f_0 , respectively, where f_0 is the fundamental frequency of the string motion.

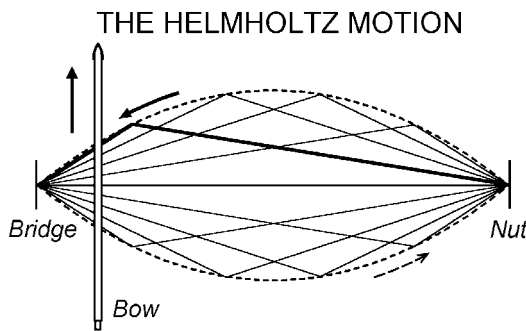


Figure T1: The string in Helmholtz motion. Every time the string’s corner passes under the bow on its way to the bridge, the string slips on the bow hair. When passing the bow again on its way to the nut, the string is captured whereafter it stays stuck for the rest of the period.

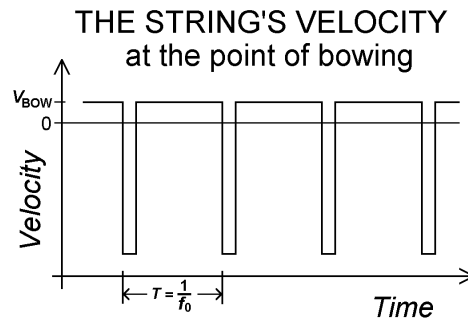


Figure T2: During the sticking interval, the string follows the velocity of the bow, v_{BOW} . In the slip interval, the string takes the velocity $v = v_{\text{BOW}} (\beta - 1)/\beta$, where β is the bow’s position relative to the string length.

A brief description of the FIDDLE simulation program

The computer program utilised for the present analyses essentially follows the concepts outlined by McIntyre and Woodhouse (1979) and Schumacher (1979), and later modifications by the same authors. Cremer (1981/1984) has given a comprehensive description of their simulation model. Computationally, the FIDDLE program is based on D’Alembert’s solution to the wave equation (described in Appendix A of Paper IV). With the exception of bow compliance and bow-hair/string friction, all losses are concentrated to the string terminations where also resonances can be programmed as part of the reflection functions. The string may be excited both in the transverse and the rotational plane.

SIMULATION MODEL OF THE BOWED STRING

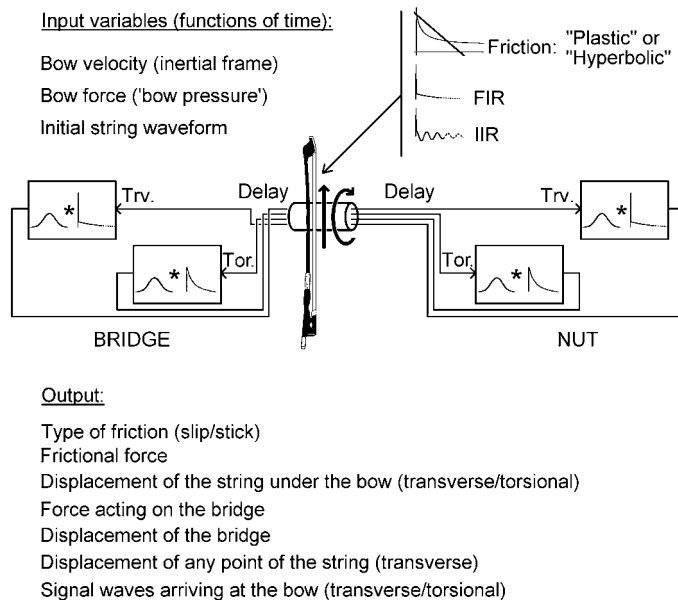


Figure T3: Symbolic signal-flow diagram of the FIDDLE program. The string is represented by a $4 \times n$ matrix, in which each element expresses a displacement from equilibrium while the columns indicate string positions (i.e., following the wave equation of D'Alembert). After each time step all elements are shifted one column within the row. The boxes symbolise reflection functions (with respect to partial velocity), which each may be composed of two convolution integrals for computational efficiency (see text). Similar functions are available for the bow.

Figure T3 (bottom) lists the output variables readily available after a simulation where detailed information is required. For the analyses of Papers IV and V, information on the signal waves arriving at the bow was crucial, and the reflection functions were chosen to facilitate identification of their origin with respect to the stick/slip pattern. For producing plots of attack quality in a bowing-parameter space (such as Fig. 7 of Paper IV), only information on the “type of friction” (i.e., slip or stick) is kept, as this makes the program run several times faster even though all the information listed is still calculated internally. All calculations are carried out in double precision.

In the boxes drawn in Figure T3, two reflection functions are indicated for each partial string end. The first one can be given any shape (as FIR), and is convolved with the signal in the traditional way in the time domain. The second function consists of a programmable Dirac delta followed by an exponentially decaying tail, the computation of which is very simple and requires three numerical operations only. This function is convenient when wanting to reduce Q-values for frequencies lower than the string’s third of fourth harmonic (although with the expense of a negligible phase shift). The function applied alone, however, gives Q-values that increase with the harmonic numbers, due to its Dirac delta.

The first function typically takes one of the three following shapes, with or without low-pass filtering: (1) gaussian; (2) “constant-Q function” (Woodhouse and Loach 1999), or (3) “string stiffness function” based on the Airy function (Woodhouse 1993).

Friction models

Two friction models are available: The “hyperbolic” friction function, which gives a friction-coefficient curve of the form

$$\mu = c_1 + c_2 / (v_{\text{relative}} + c_3),$$

where

c_{1-3} are programmable constants, and v_{relative} is the relative speed between the bow hair and the string surface. The curve is uniquely defined with three coordinates, and is multiplied with the normal bow force in order to define friction force as function of relative speed.

At the uppermost plot of Figure T3 a movable “load line” is sloping from “potential friction force” (explained next page) at the ordinate, to ditto divided by the string’s impulse surface impedance, at the abscissa. Whenever this line surpasses the limiting static frictional force at the ordinate, the friction becomes dynamic (sliding). When the load line intersects the friction curve in two points, the seeming ambiguity is resolved by evaluating the friction’s history (continuing the status of the preceding time step). As soon as the load line falls below the curve, the friction becomes static. (See e.g., McIntyre and Woodhouse 1979, pp 98-99.)

The “plastic” friction model (Smith and Woodhouse, 2000) is function of temperature at the point of contact (see Appendix D of Paper IV). It gives friction values that decrease as the temperature increases as result of energy dissipation, or increase as the temperature decays due to natural heat flow to the environment, the time constant of which is programmable. The hyperbolic curve described above can for certain models be said to approximate asymptotic friction values, i.e., results of lasting relative velocities, in the plastic friction-domain (see the middle panel of Figure T4). It hence lies in the nature of plastic friction that hysteresis occurs when the relative velocity varies.

Examples of friction trajectories during Helmholtz motion resulting from different friction models available

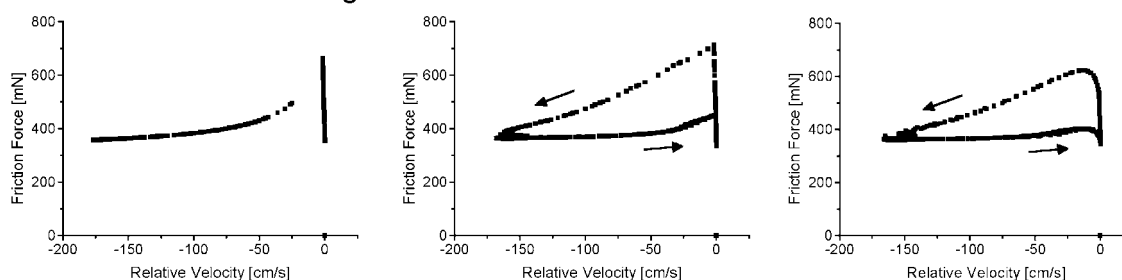


Figure T4: Examples of friction trajectories obtained through simulations with different friction models. Left panel: “hyperbolic” friction; middle and right panel: two versions of “plastic” friction. From simulations with String II of Paper IV at steady-state Helmholtz motion ($v_{\text{BOW}} = 20$ cm/s; $F_Z = 900$ mN; $\beta = 1/8$). The trajectories have noticeable influence on the string spectrum—the “hyperbolic” friction giving more energy in the upper partials than the other two models.

Special features

In addition to the numeric outputs, FIDDLE provides animated graphic information on string and bow movements. This has proven most helpful in certain cases, as for instance with the analyses of ALF of Paper V. The picture series can be stored as a graphic file for later replays without underlying calculations.

For the analyses of spiccato in Paper III a two-track recording of an impulsive excitation of a Guarneri violin was used to create a transfer function (from sound pressure and the force signal of an impulse hammer at the bridge), later to be convolved with the simulated force-on-bridge output in order to produce sound files. The same method has been utilised in the preparatory analyses of some transients, and for ALF. As an option FIDDLE gives the force-on-bridge output in 16-bit sound format.

The time-dependent input variables

The input consists of two time-dependent variables: *bow force* and *bow velocity* (inertial frame) in addition to the choice of bowing point, β . These can both be programmed through a

set of specially designed functions, including several for system analysis, e.g., “units per second”, “decibels per second”, trigonometric functions or a “chirp”. Optionally, FIDDLE provides a second β for the possibility of playing with a double bow in order to see some effects of a (quasi) hair-ribbon width. The “two bows” have a definable force distribution to simulate tilting of the bow. [This, of course, cannot compare to the method employed by Pitteroff (1995), where the bow-hair ribbon was simulated by use of the finite element method.] It is further possible to place an object (mass, spring or resistance) at an arbitrary point of the string. The latter has been utilised for analyses of what musicians refer to as “harmonics”, i.e., “flageolet tones”, the results of which are not included in this thesis. Simulations can be initiated with energy present in the string in form of travelling partial waves of arbitrary shapes. This has shown very practical for analysis of system details, as well as for simulations of pizzicato. At the conclusion of a simulation, all internal variables may be stored to file so that new runs can be made from the point where the program was halted.

Built-in tools for analysis

FIDDLE is a menu-operated program, where plots of spectra, friction trajectories, numbers of periods elapsing before Helmholtz triggering in a force-acceleration parameter space, and other analyses are immediately available at the end of each session. Spectral amplitudes of partials (harmonic or inharmonic) can be stored in memory as references to be subtracted from similar (logarithmic) amplitudes of subsequent simulations. Before initiating any simulation—after programming string impedances, propagation speeds, reflection functions, etc.—FIDDLE provides built-in routines to determine the true mode frequencies (inharmonicity), Q-values (including specified loss at each string termination), string point admittances, and a series of other details important for the design of simulations to follow.

“Potential friction force”

The information on, “waves arriving at the bow” has shown extremely important in the analysis of the relation between bowing parameters and acoustical outcome. On basis of the bow velocity and the four waves (transverse and torsional) arriving at the bow after bridge and nut reflections, respectively, a most important value termed “potential friction force” is calculated as an option. This value must surpass the limiting static frictional force in order to produce a slip, and is therefore essential in all analyses of triggering patterns.

The frictional force between a noncompliant bow and the string during the static intervals may be expressed through the following equation [which can be derived from McIntyre, Schumacher, and Woodhouse, 1983, Eqs. (B13) and (B14)]:

$$F_{ST}(t) = 2Z_{CMB} \left[v_b(t) - \sum_i^4 v_i(t) \right], \quad (T.1)$$

where

Z_{CMB} = the combined wave impedance of the string, i.e.,

for a string with rotational freedom:

$$Z_{TRV}Z_{TOR} / (Z_{TRV} + Z_{TOR}), \quad \text{otherwise: } Z_{TRV}$$

v_b = velocity of the bow

$v_i(t)$ = partial wave i , (i.e., $\partial y_i / \partial t$) arriving at the bow:

$v_1(t)$ and $v_2(t)$ = *transverse* signals propagating
away from the nut and the bridge, respectively

$v_3(t)$ and $v_4(t)$ = *torsional* signals propagating
away from the nut and the bridge, respectively.

The sum of the four partial signals (i.e., the velocities, $\partial y_i/\partial t$) gives the surface velocity the string would have taken at the point of bowing if the friction suddenly disappeared. It is convenient to refer to $F_{ST}(t)$ as “potential frictional force” regardless of whether the friction be static or not. Although explicitly referred to in Papers IV and V only, the concept of potential frictional force has been a cornerstone in all the present wave analyses.

Errata

Paper I: Two of the equations in the parameter list for Eq. (3.1) at the top of page 23 should read:

$$t_1 = 2X/C_{TRV}, \text{ and } t_2 = 2(L - X)/C_{TRV}.$$

In the caption of Figure 5, at the top of page 24, the beginning of lines number 6 and 15 should read, respectively:

$$R_4 = R_3(f_2 + f_1)/(f_2 - f_1)$$

and

$$K_8 = 2\pi f_3(R_7 + R_8)$$

Paper IV: In page 11 and the figures of page 20, the orientation of the *relative velocity* is misleading. The correct orientation is: $v_{\text{relative}} = v_{\text{string}} - v_{\text{bow}}$.

Paper V: At the top of page 9 – in Figure 1, the Q-values should be divided by a factor 2, i.e., the tick labels of the ordinate should read 5, 50, and 500, respectively.

Paper V is printed here with pages 11 through 13 in scrambled numerical—but correct informational—order.

Introduction and comments to Paper I

Acceptance limits for the duration of pre-Helmholtz transients in bowed strings

Anyone who has tried playing a string instrument has experienced the difficulty of producing bowed tones of agreeable quality. In particular, pleasant tone *onsets* are hard to achieve. As discussed in the Papers II through IV, intrinsic physical hurdles present themselves during the first part of the transient. Nonetheless, most professional string players seem to be mastering tone onsets quite well, although it may be added that attack quality is varying a lot also between these.

To simplify the issue somewhat: the onset quality is for a large part a question of making the transient leading to the Helmholtz motion as short as possible, which implies attaining periodic slip/stick triggering as early as possible in the transient. By nature bowed instruments produce stochastic noise, which certainly can be said to be part of their charm. The major source to this “noise” is the sliding of the string on the hair ribbon. If the string slips more than once per nominal period, it consequently increases the noise-to-tone ratio. In particular: if the slips are irregular, very much of the sound becomes “pitchless”, and the sonorous tone disappears (as well as the “charm”, one may assume).

Noise generation in a bowed violin string

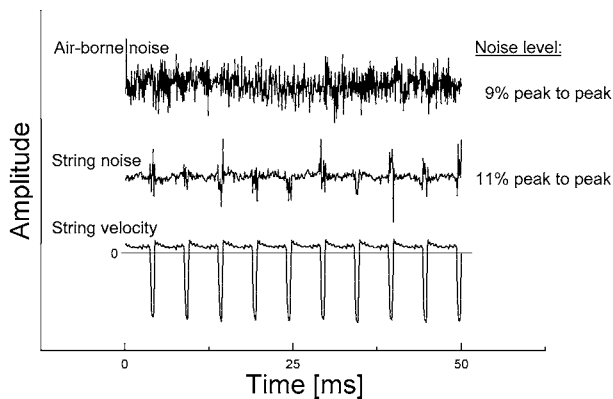


Figure T5: Noise isolated from a violin tone, and the string signal in steady-state Helmholtz motion (i.e., with regular triggering). By use of the Bluestein filter (Bluestein 1968) combined with a frequency-domain comb filter, noise can be separated from the signal without bleeding to the neighbouring elements in the time domain. The plots show that noise—although apparently continuous in the sound pressure—is mainly generated in pulses during the slipping intervals. Amplitudes up to about 10% of the signal amplitudes are typically seen.

By recording and inspecting the string-velocity waveform, one can estimate the number of nominal periods elapsing before regular periodic triggering occurs. Paper I addresses two questions: (1) How quickly are professional violinists able to establish regular triggering during practical playing? (2) From the advanced listener’s viewpoint: How much ‘off target’ can the player be before his/her attack is perceived as one of unacceptable quality? (It should be taken into consideration that any additional slip introduced during the starting transient most likely will prevail for a certain number of periods before expiring.)

Acceptance of noise in bowed “neutral” attacks

The main finding of Paper I is that bowed violin transients are normally short in terms of irregular stick-slip action. For an open violin G-string, the acceptance limit was estimated (by a panel of 20 advanced string students and professionals) to 50 ms (≤ 10 nominal periods) for

attacks with prolonged periods, and 90 ms (≤ 18 nominal periods) for multiple-slip attacks. The attacks were supposed to sound “neutral” as for practicing a scale.

When verifying the attacks of normal playing (from recordings of two violinist unaware of the subject of the research), it was found that for most musical examples the attacks were well within these limits, and that 44% (out of 1694 attacks) were found to be “perfect” (defined as less than 5 ms elapsing before the occurrence of Helmholtz triggering). The way to determine whether or not a period is “noisy” is to consider the string velocity under the bow, to see how many times it crosses the zero-velocity line during the fundamental period. A clean attack would normally¹ show two zero crossings per nominal period only.

Character of noise varying with the musical context

Naturally, the musical style will have some influence on the character of the bowed attack. For example, in the theme of Bizet’s *L’Arsienne* suite, a good part of the recorded attacks showed prolonged periods (“excess bow pressure”) giving a rougher, more “biting” sound. At the other extreme, Bach’s *Preludio V* from *Das Wohltemperierte Klavier* (consisting of rapid *détaché* scales at medium sound volume) showed long multiple-slip transients, producing a lighter, looser sound. Of the three categories “prolonged”, “perfect”, and “multiple slip”, the “perfect” attacks showed the highest rate for all bowing styles at all dynamic levels. However, the distributions on either side of this class differed substantially with the dynamics. No prolonged-periods were ever observed at soft levels (*piano*), where, on the other hand, multiple-slip transients of very long duration were sometimes seen. At higher dynamic levels (*mezzo forte* and *forte*) all the three categories were observed, but in *martellato* strokes, the first few periods were usually prolonged. Such attacks leave an impression comparable to first-letter consonants in speech.

Background for the study

In Papers II through IV, the theme is “perfect attacks” and how to achieve them. To our knowledge no report did earlier touch the issue of string waveforms during onset transients as performed by accomplished professionals, although transient spectra had been subjected to investigation by some authors (e.g., Backhaus 1932, see Figure T6, below). We found it important to establish a reference here in order to avoid emphasising phenomena of minor relevance to professional performance. The results of Paper I show clearly that string players are very sensitive to the quality of bowed starting transients, and that they are unanimously able to discriminate between onsets of small differences in the triggering pattern. The paper shows furthermore that professional violin players for a great part are able to adjust their bowing parameters for each tone individually in order to quickly establish the adequate periodicity.

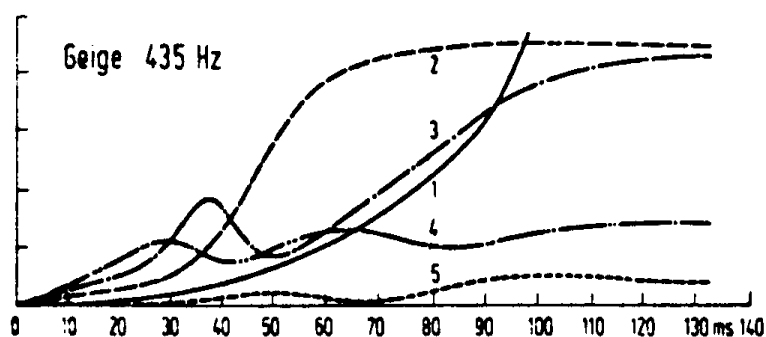


Figure T6: In the Backhaus plot (presented by Cremer, 1982) the first harmonic builds up slowly. With regular periodic triggering, it would have developed simultaneously with the others.

¹ Great torsional activity sometimes causes minor zero-line crossings of the transverse signal even during stick. Since these occur without dynamical consequences, small detours are ignored.

Figure T6 (previous page) shows the development of a violin tone (A_4). In this attack the triggering has most likely been irregular (with multiple slip) until the magnitude of the first harmonic surpasses that of the second one (i.e., after >95 ms). The musical context must determine whether this attack is to be categorised as “good” or “below average”. Judged as a “neutral tone” it is doubtful that it would have received any favourable characterisation.

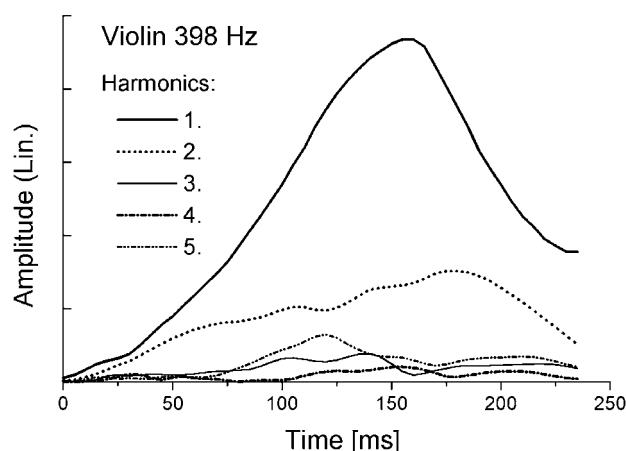


Figure T7: Violin attack from recording of Tchaikovsky Violin Concerto in D (Kyung Wha Chung, soloist). Probably an example of periodic triggering from the very start. Notice that the first harmonic is leading the build-up.

The harmonic envelopes displayed in Figure T7 were derived from a modern commercial recording with the soloist performing in a concert hall. Due to reverberation of the hall, the envelopes might have been slightly altered, but it is clearly visible that the first harmonic is the dominant partial during the entire build-up (and decay). This is a strong indication of “perfect” triggering. The pitch (G_4) is one whole step lower than the violin tone measured by Backhaus for the previous plot. In that plot the envelope of the first harmonic was not completed, so we do not know how long the transient really is. The note shown in Figure T7 was most probably executed on the G-string, and was played short and loud with a small accent. It was taken from the final movement of the concerto, and was picked from a musical phrase with a number of attacks of similar quality.

Introduction and comments to Paper II

The bowed string computer simulated – some characteristic features of the attack –

In order to understand some of the problems involved in the creation of the Helmholtz motion it is useful to investigate Figure 3 and Eq. (3.3) of Paper II. In terms of periodic triggering, the first frictional obstacle presents itself already during the first sounding period: At the time $t = t_{\text{rel-}} + T - t_1$, that is $T(1-\beta)^-$ after the first slip, a force peak threatens the bow's grip on the string (stick), which for obtaining periodicity should be lasting until time $t = t_{\text{rel-}} + T$, where the second release (slip) is supposed to happen (t_{rel} = time of the first release; T = the natural period of oscillation; t_1 = the time required for a transverse wave to propagate from bow to bridge and back; β = the relative bowing position with respect to the string length). See also Figure 1 of Paper II². (In Paper IV, the time value $t_{\text{rel-}} + T - t_1$ is referred to as **A**, while $t_{\text{rel-}} + T$ is referred to as **B**.)

There is one major difference between the friction forces during the transient and those operating at steady state: During steady state, over the whole period, the friction force is mainly determined by (i.e., taking values similar to) the value of the sliding friction, while being relative insensitive to variations in bow speed. During the transient, however, the static friction force is rapidly varying, with values directly related to the bow speed.

The resistance-reactance reflection function

Schelleng's equations on minimum and maximum "bow pressure" for a given bowing position and speed, albeit referring to string-termination *resistance* only, are in fact based on a resistance-reactance model, that is, dashpot and spring in parallel. Without the spring, the whole dynamic system would have been moving in the bow-stroke direction. Paragraph 5 of Paper II investigates the time constants' influence on playability in such models. When the dominating time constant is near the fundamental period or larger, it affects the playability in the sense that the velocity/acceleration delta becomes smaller, making the instrument "harder to play" (see Fig. 6 of Paper II). Utilising terminology from Paper IV: the difference between the frictional force at **A** and **B** is influenced by this time constant. The greater the difference, the more tolerant the system is to the bow velocity.

Bow compliance

Dynamic bow compliance (discussed in Paragraph 6 of Paper II) can be considered to represent an additional frequency-dependent admittance at the string's point of excitation. As such, the complying bow will absorb energy in frequency ranges where its admittance is comparable to the point admittance of the mounted string. On a violin, however, most string-mode frequencies will have point admittances much higher than those of the bow, ensuring a minimal energy leakage through the bow's vibrations. Simulations show that even for a heavy violin G-string, most transverse modes would exhibit point admittances much higher than the bow's peak admittances. Changing to lighter strings or bowing the same string at a larger β would give higher string point admittances yet.

² Notice: In this figure, although based on computer simulations, force peaks have for clarity been *drawn* up to the limiting static value at each stick-slip and slip-stick transition. This was done in accordance with Figure 3 of the Schelleng 1973 paper. Later it has been shown that the limiting value is never reached during realistic capture; see Smith and Woodhouse (2000).

For lower (sub-fundamental) frequencies, however, the bow has a potential of acting as an absorber, since at non-mode frequencies the string's point admittances are much lower. Figure T8 (from Guettler and Askenfelt 1995) shows the longitudinal admittance of bow hairs mounted on a bow clamped relatively rigidly at the frog (clamped with rubber around it). A handheld bow would probably give somewhat higher admittance in the lowest region. Figure T9 shows the simulated point admittance of a high-gauge violin G-string bowed at $\beta = 1/9$ for comparison.

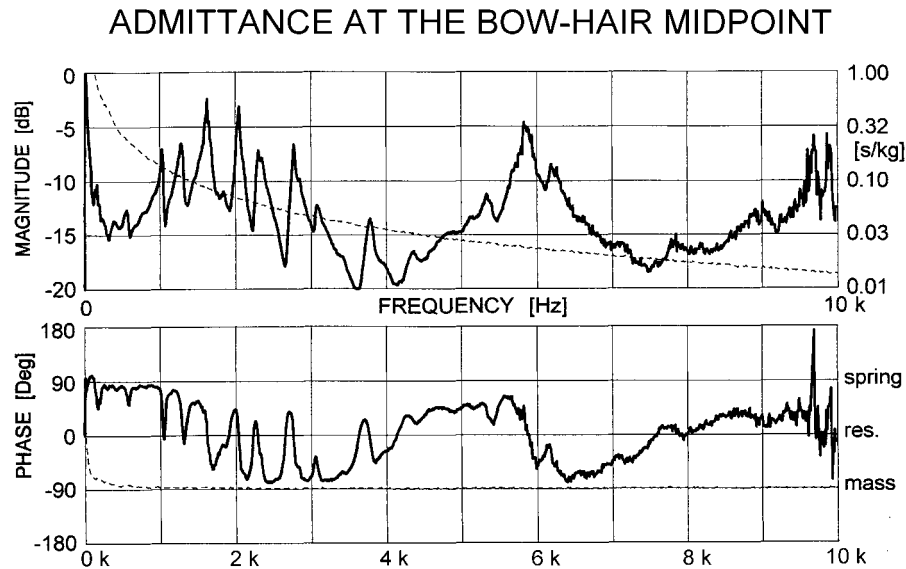


Figure T8: Bow-hair admittance in the longitudinal (i.e., the bowing) direction (from Guettler and Askenfelt 1995). For most normal modes (see Figure T9 below) a violin string's surface point admittance is much higher than the bow's admittance at any frequency. The dotted line indicates the accelerometer mass, for which the spectrum has been compensated.

STRING ADMITTANCE AT THE POINT OF BOWING in a violin G-string (simulated)

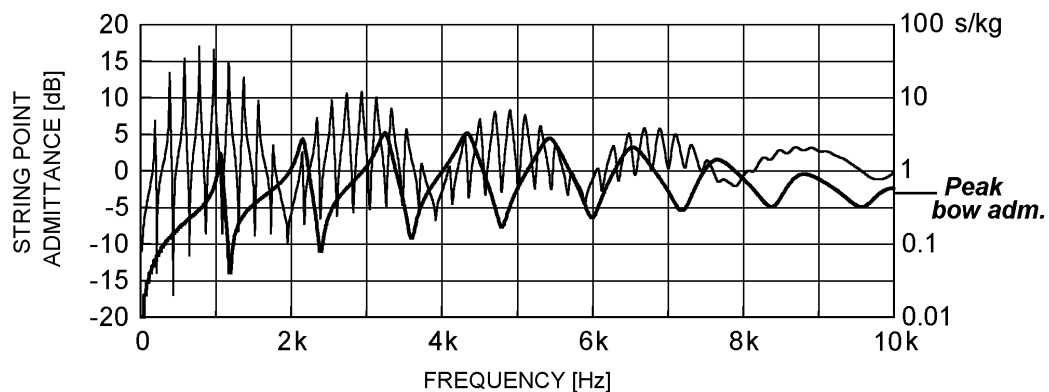


Figure T9: Simulated point admittance of a high-gauge violin G-string ($f_0 = 196$ Hz; $\beta = 1/9$; other string parameters as for String II of Paper IV, Fig. 11). Thick line: torsional admittance; thin line: transverse admittance. Of the mode frequencies, very few exhibit admittances comparable to peaks of the bow-hair admittance (≈ -3 dB). “Sub-harmonics” or other non-harmonic frequencies might, however, be approaching admittances found in the bow. The combined surface point admittance of the present string without reflections is 1.86 s/kg (= 2.7 dB).

For the examples of Figure 7 (b) and (c) in Paper II, the bow hair was given unrealistically high mobility (1.4 s/kg) in order to visualise the effect of low-frequency damping. For larger string instruments damping of this order might come more into play, though, since admittances of their strings are closer to matching those of their bows. Double bass strings normally have characteristic transverse wave impedances ranging between 1.3 and 5.3 kg/s—that is 3.5 to 14 times higher than the 0.37 kg/s of the “heavy” violin G-string above—while the mass of a double bass *bow* is only two to three times higher than the mass of a violin bow.

Moving into steady-state oscillations

Figure 8 of Paper II (repeated as Fig. T13 in this thesis) sums up much of the article’s content: The “perfect attack” starts with very strict speed/acceleration requirements. But, with those satisfied, the “Schelleng room” quickly opens, offering a great palette of timbre qualities even for a single bow-force value³. The figure is a qualitative schematic description of the bow speed tolerances with a fixed bow “pressure”. With the simple hyperbolic friction model we can write as an approximation

$$\Delta f \propto v_{B\min}(t) \propto v_{B\max}(t),$$

where Δf = difference between the limiting static friction force and the dynamic (sliding) friction force, and $v_B(t)$ is bow velocity as function of time over the entire transient.

This implies, of course, that in absolute terms a bow following the path of *minimum* bow speed when holding a high bow force, might exhibit higher speed and acceleration than a bow following the path of *maximum* bow speed with a lower bow force. Or, if preferable: the start of a firm and loud tone with *minimum* bow speed might imply higher velocities than the onset of a soft flautando with *maximum* bow speed. In practical playing, however, these two attacks would most likely be played with different β , which again influences both limits referred to.

³ In the well-known diagram where Schelleng shows the limits for maintaining the Helmholtz motion as function of position and bow force (1970, and Fig. 7 of 1973), he also indicates how bow force influences the timbre. This analysis of tone colour variation was, however, not founded on the same model as used for the calculation of the bow-force limits. Cremer, who later provided a formal explanation (1972 and 1973), described the phenomenon as “rounding of the [Helmholtz] corner”. It is worth noticing that with respect to changes of tone colour, a similar diagram might well have been drawn with *position* and *bow speed* as the two independent variables. If wanting to maintain the tone colour as much as possible during a diminuendo, the player must not only decrease the bow force, but also *decelerate* the bow.

Introduction and comments to Paper III

On the kinematics of spiccato and ricochet bowing

In order to determine which properties separate the excellent musical instrument from those of average quality it is important to possess a thorough understanding of how the instrument is played—not just at an average level, but with the technique of the highly qualified professional. The bow constitutes a good example. Properties of the violin bow have been investigated by a handful of researchers, most of them focusing the bow's natural modes (Schumacher 1975, Bissinger 1993, and Askenfelt 1995). A few reports have zoomed in on the bow's bounce properties (Abbott and Doyle 1990, Bissinger 1995, and Askenfelt and Guettler 1998), which constitute an important part of the features by which the quality of a bow is judged by professionals. However, without comprehensive knowledge on how the “off-string techniques” are executed by first-rate string players, it is hard to pinpoint the properties that make these performers claim one bow superior to another.

The character of spiccato and ricochet

The main musical reason for utilising techniques such as “spiccato” (from Italian *spiccare*: “clearly separated, cut off”) and ricochet (French: “indirectly rebounding”) is to achieve crisp and clearly separated tones of a bouncing or percussive quality, most often performed in quick succession. It lies in the nature of the bowed attack that if the slip-stick triggering shall be periodic from (near) the onset, some time is required for the string amplitude to build up, while this to some extent contradicts the combination of impulsive or percussive excitation *and* sizable amplitude. When recording the string velocity of well performed spiccato tones (as demonstrated in Figure 6 of paper III) build-up times in the order of 30 ms can typically be seen, after which time the bow leaves the string, and an exponential amplitude decay takes over. In most cases this is time enough for adequate amplitude to form in a violin. A build-up time of this magnitude does not interfere much with the impression of percussiveness.

Bow movements in spiccato

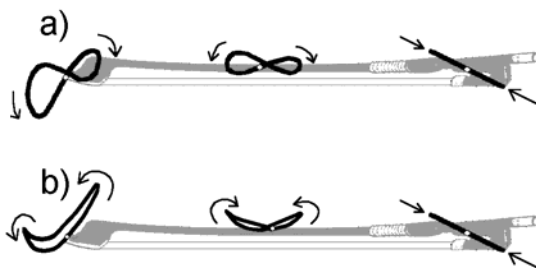


Figure T11 Different bow movements during spiccato. Only a), where the stick's trajectories describe figures of eight, will produce clean attacks. This pattern results from a combination of translational and rotational movements, the latter with twice the frequency of the former.

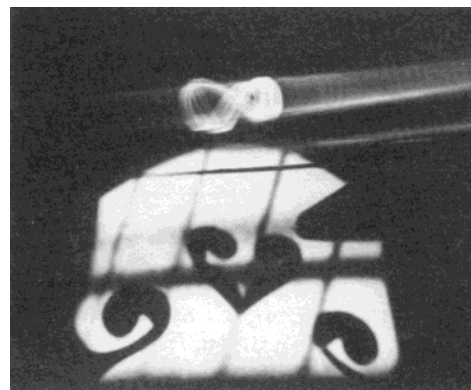


Figure T12 Photo of the bow-stick's trajectory during a successful spiccato. The bow stick's midpoint was marked with a reflective dot in order to display its trajectory (after Guettler 1992A, reprinted with permission from Yorke Edition).

The trajectories of the two bow movements

In spiccato and ricochet the bow is manipulated with a combination of translational and rotational movements. A point near the frog serves as an axis for the latter. During a ricochet stroke the axis (i.e., the frog) is moved in a more or less straight (horizontal) line in the stroke direction as long as (the rotational) bouncing takes place. In spiccato the axis is moved diagonally, as shown in Figure T11 by the frog. The translational movement is executed with half the frequency of the rotational one. In the two spiccato examples given in this figure, the only difference is the timing between the translational and the rotational movements. Only the pattern at a) produces clean attacks. Here the bow is in contact with the string when it changes direction, thus getting a good grip on the string before the first release takes place. The release happens just before the normal bow force reaches its maximum. With pattern b) the bow changes direction in the air and lands on the string with substantial horizontal speed, causing multiple slips to occur at every onset.

The four phases of spiccato

With pattern a), after (1) a quick tone build-up, the bow bounces off the string, which then experiences (2) an exponential decay due to the natural damping of the system. Thereafter, two more phases are seen to be preparing the string for the new stroke in the opposite direction: (3) a “forced decay”, where a returning, lightly touching bow (without disturbing the triggering pattern) provides quicker damping than would be caused by the system’s natural losses alone; and finally, as the bow force increases: (4) a sudden firm grip on the string, quickly restraining the remainder of the Helmholtz corner, preventing it from interfering with the creation of a new pattern with reversed rotational orientation.

The kinematical difference between ricochet and spiccato

The fourth phase described above (i.e., the bow holding the string in a firm grip in order to quieten it) points to one substantial difference between *spiccato* and *ricochet*. Whereas spiccato strokes are performed with alternating bowing directions, implying that the Helmholtz-corner rotation has to change orientation for each new stroke, a series of ricochet strokes is performed with all strokes in the same direction, i.e., “in one throw”, which means that the string’s fading wave pattern at all times can be “refreshed” without prior damping. (A good illustration of the difference is found in Figure 10 of Paper III.) Repeated high-quality ricochet strokes can hence be executed at a faster rate than comparable strokes performed as spiccato. Otherwise, from a musical point of view, their qualities are often very similar.

The bounce rate

The natural bounce rate of a bow rotating with a fixed axis through the frog can theoretically be estimated to be (see Askenfelt and Guettler 1998)

$$f_{\text{BNC}} = \frac{1}{2\pi} \sqrt{\frac{T_{\text{hair}} \left(\frac{r_s}{1 - r_s/L_{\text{bow}}} \right)}{J_x}}, \quad (\text{T.2})$$

where

T_{hair} = tension of hair

r_s = distance from the pivoting point to the impact against a rigid support (quasi string)

L_{bow} = (free) hair length

J_x = moment of inertia.

Equation (T.2) compares well to practical measurements for most of the hair length, but gives too high frequencies for impacts farther than 2/3 of the hair length away from the frog. The measured bouncing rate for a violin bow is typically between 6 and 35 Hz for impacts at hair lengths of 8 to 60 cm from the frog when the hair tension is 60 N. Q-values were found in the order of 30 to 50. Equation (T.2) is, however, only valid as long as the bow hair is in direct contact with the support. When bouncing “off the string”, the bouncing rate is dependent on gravity and/or an index finger providing the “downward” force. With gravity alone, this force is constant, so the bouncing rate will increase as the time interval “off string” decreases—just like a ball bouncing on the floor.

Adjustment of the bounce rate

For the player, in addition to adjusting the bow hair tension, there are four manoeuvrable parameters that influence the bounce rate:

- (1) the firmness of the bow grip (i.e., the “downward reactive force”);
- (2) the point of impact along the hair [as expressed in Equation (T.2) above];
- (3) the distance from the bridge (the string getting more compliant as one moves away from the bridge);
- (4) the tilting angle of the hair ribbon (the compliance of the hair ribbon increasing with the angle).

In a ricochet stroke one or more of these factors have to be continuously adjusted in order to keep the bow bouncing at a uniform rate as the point of impact is moved along the bow hair.

After a series of ricochet tones, the music often requires sustained tones, implying that further bobbing must be quickly restrained. The way to do it is not to grip the bow tightly, as this would just increase “downward reactive forces” and thus only serve to raise the bounce frequency. Before suggesting a solution to the problem, the concept of “point of percussion” needs to be introduced.

The point of percussion

In an object rotating around a fixed axis, a point can be defined as “the point of percussion” (PoP): When hanging a bow vertically with the fixed axis through the frog, PoP is found where the bow has its “effective pendulum length”, i.e., L' of the following equation:

$$\tau = \frac{2\pi}{\omega} = \sqrt{\frac{L'}{g}} \quad (\text{T.3})$$

where

τ = period of oscillation

$L' = I/(M\bar{r})$ = effective pendulum length

I = moment of inertia

M = mass of object

\bar{r} = distance from axis to center of gravity.

g = acceleration of gravity

(In a tennis racket or a baseball bat, PoP is often referred to as “the sweet spot”. For a straight rod of length L with uniformly distributed mass and the rotational axis in one end, PoP is found at a distance $2/3 L$ from the axis.)

When hitting the string at the frog side of PoP—where the string in effect is establishing a new axis—conservation of the angular momentum makes the bow seek to maintain its rotational orientation with respect to the string. That drives the frog upwards, as shown in Figure T10. When hitting the string on the tip side of PoP, the rotational orientation gets reversed, forcing the frog downwards, while hitting the string directly at PoP leads all energy into the string so little or no reaction is noticed at the frog.

Preventing the bow from further bouncing

To stop the bow from further bouncing, one should therefore, considering the analysis above, (partly) give in to the movements of the frog, much like playing a ball “dead” by complying to its movement, whereby an appreciable amount of the energy is absorbed.

RICOCHE:

Reactions at the frog after string impact

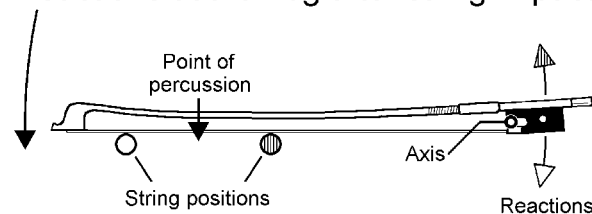


Figure T10: When hitting the string at a point outside of the point of percussion, a downward impulsive force is felt at the frog. If hitting the string inside of the point of percussion, an upward force is felt. By “giving in” to the frog reactions, further bouncing of the bow can be effectively restrained.

In addition to this action, it may also help to tilt the bow-hair ribbon (or increase its tilting angle if already present), for in that way to lower the bounce frequency. This is particularly useful when ending the ricochet series near the bow’s head in preparation for a soft or medium-soft tone to follow. In most cases, however, the stick will continue to wobble somewhat (see Fig. 9 of Paper III), but as long as the bow hair is kept in touch with the string, and the normal bow force is fluctuating within reasonable values, the resulting small variations in timbre will pass unnoticed.

Introduction and comments to Paper IV

On the creation of the Helmholtz motion in the bowed string

While Papers II and III seek to explain certain details concerning bowed-string transients, Paper IV tries to uncover the underlying structure of quality attacks in general. The method used resembles the method for analysing steady-state oscillations employed by Schelleng in his papers from 1970 and 1973. Quite a few years earlier Raman (1918) had discovered that in a string model with purely resistive string terminations the friction force would be describing a cyclic pattern when the string was driven in Helmholtz mode. In particular, when the bowing position divided the string in an integer ratio, the friction force would have its lowest value during the slipping interval, and its maximum value at the middle part of the sticking interval. The peak-to-peak value of this cyclic variation could be estimated to be (explicitly given by Cremer in 1981/1984, with reference to Schumacher 1979):

$$\Delta f = v_b \frac{Z^2}{R} \left(\frac{L}{x} \right)^2, \quad (L/x = \text{integer}) \quad (\text{T.4})$$

where

v_b = bow speed

x = bowing position

L = string length

Z = characteristic wave resistance of the string

R = string - termination impedance (resistive).

Schelleng (1970) had realised that the Helmholtz motion consequently could be maintained only if the value of the limiting static frictional force was Δf higher than the value of sliding friction. If not, a second slip would be introduced, and the Helmholtz pattern destroyed. On the other hand, the limiting value should not be higher than permitting a string release when the kink arrived (in other words: not higher than the “potential friction force” following immediately after it).

Applying Schelleng’s method for analysis of the transient

During an onset transient the situation is comparable, although here the variations of friction force appear to be considerably more complex. It is, however, possible to localise four critical points in the transient where regular triggering is particularly endangered when regarding a simple bowed-string model. In Paper IV these points of time are labelled **A**, **B**, **C**, and **D**. Since the friction force during the transient is strongly related to the bow speed, it is thus possible, for a given “bow pressure”, to calculate approximate values of “acceptable” velocity or acceleration with reference to these four points of time.

The increasing acceptance of bow speed during the transient

Figure T13 shows schematically how the range of acceptable bow velocities is growing in the course of the transient. By utilising equations from Paper IV, we can get a rough numerical estimate of this development: While the bow-speed tolerance ratio for a given bow force indicated by Eq. (8v) is $v_{0\max}/v_{0\min} \approx 1/(1-2\beta)$ around the second slip, Schelleng’s equation [Eq. (1) of the same paper] indicates the ratio $v_{\max}/v_{\min} \approx 2\beta r/Z$, ($\beta \geq Z/2r$) at steady state. By

replacing r/Z with a realistic figure, say one hundred, we get an indication of the ratio of tolerance after the transient has expired: $r/Z = 100$ gives $v_{\max}/v_{\min} \approx 200\beta$. For the arbitrarily chosen bowing position $\beta = 1/8$, this is a factor 18.75 higher than the ratio $v_{0\max}/v_{0\min}$ at the outset of the transient.

The value “18.75” should of course not be taken too literally, as there are many other factors that would enter the equation: torsion, width of bow-hair, non-linear string parameters, to name but a few. Anyhow, the magnitude gives an indication of how drastically conditions are changing during a relatively short-lasting transient.

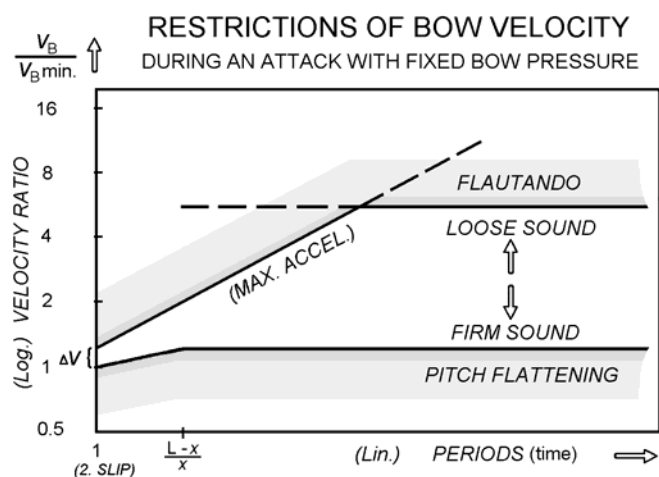


Figure T13: Qualitative schematic illustration of how the bow-velocity tolerances increase with time in the course of the transient. The absolute ranges of acceptable velocities are also functions of the bowing position and the friction delta.

Notice: In Paper II, from which this figure was copied, the ratio $v_{B\max}/v_{B\min}$ at the second slip was reported to be “typically near” $1/(1 - \beta)$ for strings with torsion and “normal” damping, while in Paper IV it was calculated to $1/(1 - 2\beta)$ for a nearly loss-free string without torsion, provided $1/\beta$ was an integer. The information value of these expressions lies mainly in how β affects the ranges of “acceptable” velocity—not only at steady state, but during all parts of the bow stroke.

The problem of including torsion in the equations—“torsionally related ringing”

The equations of Paper IV refer to simple systems without torsion. When a string flyback takes place, torsion will cause the system to ring. For this reason the equations of Paper IV are not immediately applicable for complex systems, even though it is straightforward to implement torsion in the function that approximately describes the friction-force build-up until the first slip [i.e., Eq. (3) of Paper IV].

When comparing Figure 12 of Paper IV to Figure 10 of the same paper, one striking difference is the greater tolerance for bow acceleration present at low β (i.e., at the β values $1/12$ and $1/11$) in the more complex systems of Fig. 12. At these β values tolerances of acceleration appear to be much greater than could have been expected from the equations and simulations of Fig. 10. This phenomenon might, however, be traced back to a small force peak caused by *torsional activity* around $t = t_{\text{rel}} + T$, i.e., when the second slip should happen. Utilising analyses from our next paper, Paper V, we can describe the development as follows:

Figure 4 of Paper V shows the partial responses to a unit square pulse (transverse partial velocity propagating from the nut) arriving at the bow at $t = 0$. [This can be compared to a first slip pulse, once reflected at the nut and arriving at the bow at the time $t = t_{\text{rel}} + T(1 - \beta)$.] For convenience we choose $1/\zeta$ to be an integer (ζ being the ratio between transverse and

torsional propagating speeds). The width of the square pulse shown in the figure is $T\beta$, and there is no other energy present in the string. Torsional reflections from the nut are ignored in the time interval we investigate. Different from Fig. 4 of Paper V we will here analyse a system with no other loss than what is introduced by the reflections/transmissions at the fixed bow. With these presumptions we get the following partial signals ($\partial y_i / \partial t$):

In the interval $0 < t < T\beta\zeta$, arriving at the bow from the bridge and the nut, respectively:
Torsion = 0, Transverse = Unit.

In the interval $T\beta\zeta < t < 2T\beta\zeta$:
Torsion = Unit $Z_{TRV} / (Z_{TOR} + Z_{TRV})$, Transverse = Unit,
(where Z_{TOR} and Z_{TRV} are characteristic torsional and transverse wave resistances, respectively).

In the interval $2T\beta\zeta < t < T\beta$:
Torsion (envelope) = Unit $(Z_{TRV} / Z_{TOR}) [1 - \exp(-t/\tau)]$, Transverse = Unit,
(where $\tau = T\beta\zeta / \ln[(Z_{TOR} + Z_{TRV}) / Z_{TRV}]$ —see Fig. 11 of Paper V).

At this point of time the square pulse expires. The following signals both arrive at the bow after reflections at the bridge:

In the interval $T\beta < t < T(\beta + \beta\zeta)$:
With $[1 - \exp(-t/\tau)]$ approaching unity, we get
Torsion \approx Unit (Z_{TRV} / Z_{TOR}) , Transverse = -Unit $Z_{TRV} / (Z_{TOR} + Z_{TRV})$.

Compared to a string without torsion this leads to a *reduction* of the friction force equal to
 $\Delta f_1 = [2Z_{TOR} Z_{TRV} / (Z_{TOR} + Z_{TRV})] \text{Unit} [-(Z_{TRV} / Z_{TOR}) + Z_{TRV} / (Z_{TOR} + Z_{TRV})]$ (T.5)
(where the first square bracket contains the surface point impulse impedance of the string).

In the interval $T(\beta + \beta\zeta) < t < T(\beta + 2\beta\zeta)$:
Torsion = Unit $[(Z_{TRV} / Z_{TOR}) - Z_{TRV} / (Z_{TOR} + Z_{TRV})] Z_{TRV} / (Z_{TOR} + Z_{TRV}) =$
Unit $Z_{TRV}^3 / [Z_{TOR}(Z_{TOR} + Z_{TRV})^2]$.
Transverse = -Unit $[(Z_{TRV} / Z_{TOR}) - Z_{TRV} / (Z_{TOR} + Z_{TRV})] Z_{TOR} / (Z_{TOR} + Z_{TRV}) =$
-Unit $Z_{TRV}^2 / (Z_{TOR} + Z_{TRV})^2$.

This leads to an *increase* of the friction force equal to

$$\Delta f_2 = [2Z_{TOR} Z_{TRV} / (Z_{TOR} + Z_{TRV})] \text{Unit} \{-Z_{TRV}^3 / [Z_{TOR}(Z_{TOR} + Z_{TRV})^2] + Z_{TRV}^2 / (Z_{TOR} + Z_{TRV})^2\} =$$

$$2 \text{Unit} (Z_{TOR} - Z_{TRV}) Z_{TRV}^3 / (Z_{TOR} + Z_{TRV})^3. \quad (\text{T.6})$$

With respect to our discussion above on greater tolerance of acceleration, a comparable raise of the friction force would thus happen near $t_{rel} + T(1 + \beta\zeta) \leq t < t_{rel} + T(1 + 2\beta\zeta)$, i.e., right after “**B**”. So, if triggering fails at **B**—*provided the product $\beta\zeta$ is small enough*—this peak might trigger a release just in time for the periodicity not to be critically disturbed. The effect is most clearly seen in Figures 5 and 6 of Paper V, and Figure 6 (b) of Paper II, where a small friction-force peak is rising shortly after the expiration of the major force notch. Here the subsequent “ringing” is also seen, partly due to torsional reflections arriving from the nut. Notice from the Equations (T.5) and (T.6) above: with $Z_{TOR} = \infty$, no modulation of the friction force would have occurred. (In the analysis above, Z_{TOR} is presumed larger than Z_{TRV} .)

Introduction and comments to Paper V

Wave analysis of a string bowed to anomalous low frequencies

The Helmholtz motion is far from the only steady-state wave pattern that can exist in the bowed string. Many stable patterns can develop where more than one slipping interval occur during each nominal period, in which cases the fundamental frequency and certain partials will appear suppressed or reduced in magnitude. “Ponticello” is an example of this effect. There is, however, another category of stable wave patterns that occasionally has been utilised by composers and performers (e.g., Østergaard 1999, and Kimura 1996), based on a delay in the triggering of the string release after each stick (static-friction) interval.

Triggering delay

If the normal bow force (F_Z) surpasses the Schelleng maximum [see Eq. (T.7)], the Helmholtz corner will be reflected at the bow, and, instead of causing a release, make one more turn back to the nut. In the meantime the static frictional force is building up due to repeated reflections on the bridge side. When the (rejected and reflected) Helmholtz corner meets the bow again, although most likely reduced in energy, it may, combined with the general raise in frictional force, cause the necessary triggering of release. If triggering fails again, this cycle repeats until the combined forces surpass the limiting static friction.

$$F_{Z\max} = \frac{Z \cdot v_b}{(\mu_s - \mu_d)\beta}, \quad (\text{T.7})$$

where

μ_s and μ_d are the limiting static, and the dynamic (sliding) frictional forces, respectively

β = the bowing position relative to the string length

Z = the characteristic wave resistance of the string

v_b = bow velocity, and r = resistance at the bridge (representing all losses).

The “echoes” of a string flyback

In order to achieve a periodic triggering it is necessary to have a discontinuity, or marked change, in the *signal velocity* (i.e., the time derivative of the string’s surface displacement). The greatest changes of this kind have their origin in the velocity pulse caused by the sudden string flyback. Such pulses are transmitted both as transverse and torsional signals, and either can potentially cause a string release after having been reflected at the nut (i.e., the most remote string termination as seen by the bow). Figures 5 and 6 of Paper V show these pulse reflections superimposed on a (force) ramp caused by repeated bridge reflections, or—if thinking statically—by the string’s increasing angle on the bridge side, with respect to equilibrium.

Transverse and torsional triggering

As can be seen from Figures 5 and 6 of Paper V, a force peak (caused by a torsional pulse reflected at the nut) occurs at a time $\approx T_0\zeta$ after the normal triggering time $t = t_{\text{rel}} + T_0$. (T_0 = the fundamental period, $\zeta = C_{\text{TRV}}/C_{\text{TOR}}$, i.e., the ratio of transverse and torsional propagating speeds.) Normally, the positive flank seen on the right side of the first force notch would trigger a release. If that one fails—and the string release is triggered by the above-mentioned

torsional pulse—the period gets extended from T_0 to $T_0(1 + \zeta)$. For most violin G-strings, ζ is close to 0.2, implying that the pitch will fall a minor third.

In the case of “transverse triggering”, also the torsional pulse is too weak to cause a slip, leaving the triggering to the next *transverse* pulse arriving from the nut at $t = T_0(1 + t_1)$, or one that follows later. Figure T14 shows the most common signal paths, while the equations below give idealised descriptions of the two sets of anomalous low frequencies potentially obtainable through delayed triggering:

$$f_{ALF\ tor} \approx \frac{1}{T_0[1 + \zeta + n(1 - \beta)]} \quad (n = 0, 1, 2, \dots) \quad (\text{torsional triggering}) \quad (\text{T.8})$$

and

$$f_{ALF\ trv} \approx \frac{1}{T_0[1 + n(1 - \beta)]} \quad (n = 1, 2, 3, \dots) \quad (\text{transverse triggering}), \quad (\text{T.9})$$

where $\zeta = C_{TOR} / C_{TRV}$, i.e., ratio of propagating speeds.

It is interesting to notice that most of these frequencies are dependent on the *position* of the bow (i.e., all frequencies where $n \neq 0$ in the above equations), a fact that is most easily understood by regarding the wave paths in Figure T14. The insensitivity to bowing position for the highest torsional f_{ALF} is related to a complicated transverse-torsional transform mechanism that implies reflections of torsional waves at the bridge-side string termination too. Figure 4 of Paper V and the related text seek to elucidate this⁴.

Paths of triggering waves

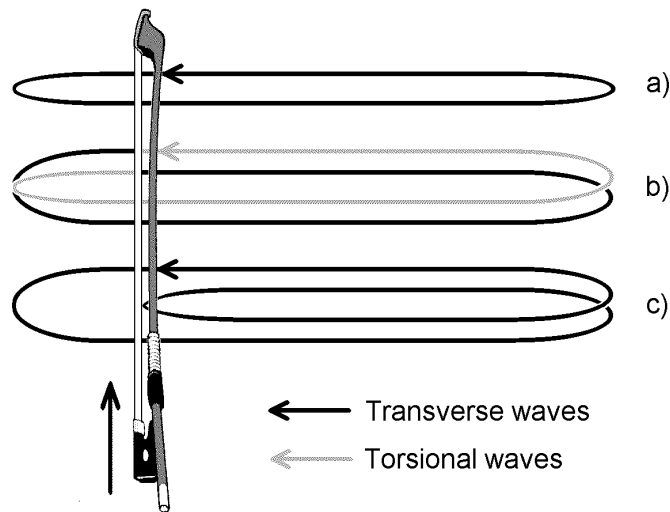


Figure T14:

Basic signal paths for

- a) Helmholtz motion (i.e., triggering at mT_0) ($m = 0, 1, 2, \dots$)
- b) Torsional triggering (at $mT_0(1 + \zeta)$)
- c) Transverse triggering (at $mT_0[1 + (1 - \zeta)]$).

⁴ The reader should not be confused by the fact that the expressions t_1 and t_2 of Papers II and V do not have corresponding definitions. In paper V, $t_1 = T_0(1 - \beta)$, and $t_2 = T_0\beta$, while these were reversed in Paper II.

Other signal paths are possible, but most likely as combinations of a) and one or more of the extra loops of b) and c) of Figure T14. The arrowheads indicate the position of the triggering waves just before the string release.

Spectra of the ALF tones

Hanson et al. (1994) report weak or missing lower partials in sound recordings of ALF tones played on a violin (the microphone placed 0.5 to 1.0 metres away from the instrument). These missing partials are, however, not lacking energy in the string signals, which exhibit complete sets of harmonics. The fact that some lower partials disappear in sound-pressure recordings is just a confirmation of the instrument's poor radiation of frequencies below its lowest air mode (about 270 Hz), and does not affect the listener's perception of the low pitch itself.

Additional information on Paper I:

Paper II was initiated by the first author (K.G.) in order to verify or disprove the (long) duration of transients reported as “normal” in several earlier papers on the bowed string. Although most parts of Paper I were developed and evaluated in cooperation, the work was roughly split between the two authors in the following manner: The second author (A.A.) was responsible for the sound recordings of all tests, and for the instruction, classification, and statistical analysis of *the playing test*, while K.G. was responsible for the design of *the listening test*, and for selecting, filtering, and editing its sound examples. Furthermore, K.G. performed all tasks related to the testing itself, including panel presentation and statistical analyses.

Additional information on Paper III:

Paper III is a kinematic study of a bowing pattern that had already been described by K. Guettler (1992A). The second author (A.A.) was responsible for the recordings of all tests, including video recordings of a bow with light-emitting diodes attached to it. Together, A.A. and K.G. were interpreting (twice integrated) recordings of accelerometers attached to different parts of the bow during spiccato. K.G. did all the simulations—some of which were transferred to sound files—and all the analyses that followed.

How it started

Acknowledgments

My interest for bowed-string acoustics can be dated back to the summer 1968 when I attended a series of lectures on this topic given by William Fry at the University of Wisconsin, as part of a double bass conference. Not only did these lectures make me more conscious on how I played my bass (at that time member of the Oslo Philharmonic), they also right away enabled me to improve the response of my instrument by modifying its bridge (as well as bridges for many of my double bass colleagues).

The “breakthrough”, however, came with Lothar Cremer’s book “The Physics of the Violin”, which in 1986 I accidentally found it in a bookstore among literature on music—Thank you Nordli, for leaving it on that shelf! Since I had acquired an IBM PC a couple of years earlier, the idea of making a graphical program showing my students how the string was moving under the bow, seemed not so far away. With the poor calculating powers of those days’ PCs, this idea turned out not to be a very good one, though. Being somewhat more patient than my pupils, I alternatively thought of improving the program so that it could be used for calculating the effects of different bow strokes instead. Maybe some secrets were hidden there...

Of the many qualified people who have helped me in realising this project, I am particularly indebted to:

Asbjørn Krokstad, for taking me seriously, and for helping me out with technical details when I was writing my first “scientific article” to be accepted; Robert T. Schumacher and James Woodhouse, for always answering questions when I needed their expertise, and for the many enlightening discussions and suggestions they offered; Roger Hanson for introducing the phenomenon of Anomalous Low Frequencies to me, and for providing all the empiric material needed for further analyses; Johan Sundberg, for opening the doors to KTH, with its fine laboratory and all the inspiring and generous people who surround it, thank you everyone! And—maybe most of all—Anders Askenfelt, who in 1993 took a first initiative to several joint research projects to follow, and who—due to my lack of acoustical schooling—was forced to serve as my personal tutor, continuously informing me on basic acoustical concepts.

I want to express gratitude to the board at Kungl. Tekniska Högskolan that four years ago accepted me for doctoral studies even though I was a “school drop-out” with no matriculation degree, nor prior relevant formal education, just experience as a musician and a great interest for string acoustics. Honour should fall to my two supervisors, Erik Jansson and Johan Liljencrants, who on this meagre background were able to guide me through the mysteries of physics, hearing, statistics, and digital signal analysis, and much more.

I am also much indebted to my employer, the Norwegian Academy of Music, for their continuous support of my research, including a year’s sabbatical for the present work to be concluded. And to the Wenner-Green Foundation as well as the Swedish Natural Science Research Council (NFR) for providing further financial support. Without their generous help this work would never have been moving.

Finally, I must thank the IBM scientific centres of Madrid and Winchester, who created a preliminary version of the unique and unbeatably concise (but, unfortunately, *never-to-be-released!!!*) programming language APL2 for PC, through which the FIDDLE program was developed. By giving me the opportunity to follow all whims with just a few keystrokes, they made it possible for me to explore many more paths than I ever had known existed.

Knut Guettler
Oslo, April 6. 2002.

References

- Abbott, J. A. and Doyle, E. (1990) Violin bow dynamics—An initial response to John Bolander's plea. *Catgut Acoust. Soc. Journal* Vol. 1, No. 5 (Series II), 30-31
- Askenfelt, A. (1989) Measurements of the bowing parameters in violin playing. II: Bow-bridge distance, dynamic range, and limits of bow force, *J. Acoust. Soc. Amer.* 86, 503-516.
- Askenfelt, A. (1995) Observations on the violin bow and the interaction with the string. *Proc. ISMA '95*, 197-212.
- Askenfelt, A., and Guettler, K. (1998) "The bouncing bow – An experimental study" *Catgut Acoust. Soc. Journal* Vol. 3, No 6 (II), 3-8.
- Backhaus, H. (1932) *Zeitschrift für Technische Physik*. 13:31. (Illustration reprinted in Cremer 1982)
- Bissinger, G. (1993) Merging microphone and accelerometer hammer-impact modal analysis measurements: Working example—the violin bow. *Proceedings of the 11th International Modal Analysis Conference*, 850-854.
- Bissinger, G. (1995) Bounce tests, modal analysis, and the playing qualities of the violin bow. *Catgut Acoust. Soc. Journal* Vol. 2, 8 (II) pp 17-22.
- Bluestein, L. I., (1968) A linear filtering approach to the computation of the discrete Fourier transform. *Nerem Record*. 218-219.
- Cremer, L. (1979) The absorption of "secondary correction waves" in the excitation of a string by bowing—Part II. *Catgut Acoust. Soc. Newsletter* #32, 27-32.
- Cremer, L. (1972 and 1973) The influence of "bow pressure" on the movement of a bowed string. Part I and II. NL. *Catgut Acoust. Soc.* #18 pp. 13-19 and #19 pp. 21-25.
- Cremer, L. (1982) Considerations of the duration of transients in bowed strings, *Catgut Acoust. Soc. Newsletter* #38, 13-18.
- Cremer, L. (1984) *The Physics of the Violin*, Chapter 8, pp 184-188, The MIT press (original German edition 1981 by S. Hirtzel Verlag).
- Cronhjort, A. (1992) A computer-controlled bowing machine, Speech Transmission Laboratory Quarterly Progress and Status Report, Dept. of Speech Communication and Music Acoustics, Royal institute of technology, Stockholm, STL-QPSR 2-3/92, 61-66
- Gabrielsson, A. (1979) Statistical treatment of data from listening tests on sound-reproducing systems, Karolinska Institute, Dept. of Technical Audiology, Report #TA 92.
- Guettler, K. (1992A) *A Guide to Advanced Modern Double Bass Technique*, Yorke Edition, London.
- Guettler, K. (1992B) The bowed string computer simulated—some characteristic features of the attack. *Catgut Acoust. Soc. Journal* 2, 2(II), 22-26.
- Guettler, K. (1994) Wave analysis of a string bowed to anomalous low frequencies. *Catgut Acoust. Soc. Journal* 2, 6(II), 8-14.
- Guettler, K., and Askenfelt, (1995) A., Relation between bow resonances and the spectrum of a bowed string. *Proc. ISMA '95*, 231-237.
- Guettler, K. & Askenfelt, A., (1997) Acceptance limits for the duration of pre-Helmholtz transient in bowed string attacks. *J. Acoust. Soc. Am.* Vol. 101, No 5, Pt. 1, 2903-2913.

- Guettler, K. and Askenfelt, A. (1998) On the kinematics of spiccato and ricochet bowing, *Catgut Acoust. Soc. Journal* 3, 6(II), 9-15.
- Hanson, R. J. et al. (1994) Anomalous low-pitched tones from a bowed violin string, *Catgut Acoust. Soc. Journal* 2, 6(II), 1-7.
- Helmholtz, H. F. (1862). *Die Lehre von den Tonempfindungen* [On the Sensations of Tone], Braunschweig: Vieweg. [New York 1954: Dover.]
- Kanji, G. K. (1994) *100 Statistical Tests*, Sage London.
- Kimura, M. (1996) Robert Dick and Mari Kimura: *Irrefragable Dreams* (Track 3. Triskelion) CD, GEMA RA 018.
- Kirk, R. E. (1982) *Experimental Design: Procedures for the Behavioural Sciences*, Brooks-Cole, Belmont, CA, (2nd ed.)
- Lazarus, H. and Eisenberg, P. (1969), Lecture, Physicists' Convention, Heidelberg.
- Lazarus, H. (1972) *Die Behandlung der selbsterregten Kippschwingungen der gestrichenen Saite mit Hilfe der endlichen Laplacetransformationen*. Dissertation, Der technischen Universität, Berlin.
- McIntyre, M. E. et al. (1979) On the fundamentals of bowed-string dynamics, *Acustica Vol.* 43, 93-108.
- McIntyre, M. E. et al. (1981) Aperiodicity in bowed-string motion, *Acustica Vol.* 49, 13-32.
- McIntyre, M. E., Schumacher, R. T., and Woodhouse, J., (1983) On the oscillations of musical instruments. *J. Acoust. Soc. Am.* Vol. 74 (5), 1325-1345.
- McIntyre, M. E. and Woodhouse, J. (1984) A parametric study of the bowed string: The violinist's menagerie, *Catgut Acoust. Soc. Journal* 42, 18-21.
- Melka, A. (1970). Klangeinsatz bei Musikinstrumenten. *Acustica*, 23, 108-117.
- Morset, L. H., Unpublished measurements performed in an anechoic room by use of the MLS method, described in Morset, L. H., (1999) An Investigation of Vibrational and Acoustical Properties of the Violin using MLS and TV Holography. *Proc. ASA/EAA 1999, Berlin*.
- Pickering, N. C. (1985) Physical properties of violin strings, *Catgut Acoust. Soc. Journal* 44.
- Pitteroff, R., (1995) *Contact Mechanics of the Bowed String*. (PhD dissertation) University of Cambridge.
- Raman, C. V., (1918) On the mechanical theory of the vibrations of bowed strings and of musical instruments of the violin family, with experimental verification of the results. Part I. *Indian Assoc. for the Cultivation of Science*, Bull 15, 1-158.
- Raman, C. V. (1920-21) On the mechanical theory of the vibrations of bowed strings and of musical instruments of the violin family, with experimental verifications of the results: Part II. – Experiments with mechanically played violins, *Proc. Indian Assoc. for Cultivation of Science*, 6. 19-36.
- Schelleng, J. C. (1970) Pressure on the bowed string, *Catgut Acoust. Soc. Newsletter* #13, 24-27.
- Schelleng, J. C. (1973) The bowed string and the player, *J. Acoust. Soc. Am.* 53(1), 26-41.
- Schumacher, R. T. (1975) Some aspects of the bow. *Catgut Acoust. Soc. Newsletter* # 24, 5-8.
- Schumacher, R. T. (1979) Self-sustained oscillations of the bowed string, *Acustica* 43, 109-120.
- Schumacher, R. T. (1986) Measurements of the parameters of bowing, *Proc. of the 12th International Congress of Acoustics*, Toronto.

- Serafin, S., et al. (1999) An investigation of the impact of torsion waves and friction characteristics on the playability of virtual bowed strings. *Proc. 1999 IEEE Workshop on Applications of Signal Processing to Audio and Acoustics*.
- Smith, J. H. and Woodhouse, J. (2000) The tribology of rosin. *Journal of the Mechanics and Physics of Solids*, 48, 1633-1681
- Woodhouse, J., (1993) On the playability of violins. Part I: Reflection functions. *Acustica* Vol. 78, 25-136.
- Woodhouse, J., & Loach, A. R., (1999) Torsional behaviour of cello strings. *Acustica* Vol. 85, 34-740.
- Woodhouse, J., Schumacher, R.T., and Garoff, S. (2000) Reconstruction of bowing point friction force in a bowed string. *J. Acoust. Soc. Amer.* 108 (1), 357-368.
- Winer, B. J. (1971) *Statistical Principles in Experimental Design*. McGraw-Hill, New York (2nd ed.).
- Østergaard, E. (1999) *Ritus* (composition for unaccompanied viola). Norwegian Music Information Centre.