Some changes of the spectrum caused by pitch flattening of the bowed string

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Abstract

The phenomenon of pitch flattening of a string bowed with excess bow force ("bow pressure") is well documented[1 2 3] and is related to the hysteresis function of the resin's friction curve. One consequence of such a frequency shift is the very noticeable change in the spectral envelope, which cannot alone be traced back to the changes in the spectrum of the "input signal", i.e.: the velocity of the string at bowing point. This paper discusses the highly resonant nature of the transfer function between velocities of the bridge and the string at the point of excitation, and illustrates the effects through computer simulations.

Slipping period versus pitch flattening

Simulations performed by the author show that during a stroke with increasing bow force, the slipping period typically approaches an asymptotic minimum value of $t_{SLIP}=T_0x/L$ during the expiration of the "rounded corner"[4] - and holds this value also when pitch flattening extends the total period between each slip. (T_0 = natural period of the string; x = distance between bridge and the point of excitation; L = total string length. See fig. 1:)

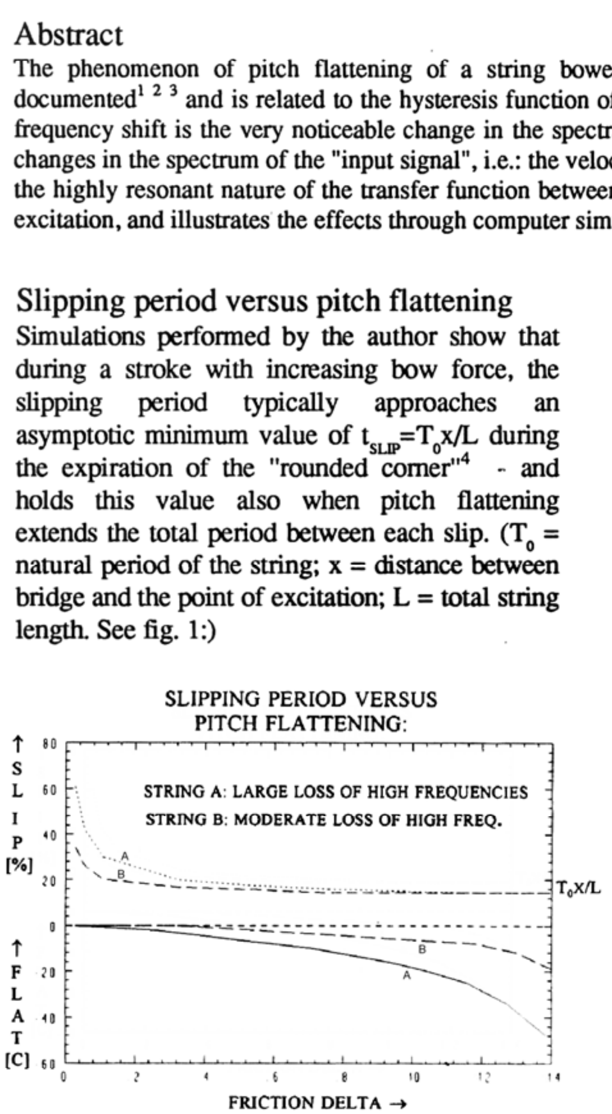

Figure 1: Upper half - slipping period in percentage of the natural oscillating period (T_0) of the system. Lower half - pitch flattening in cent. Abscissa indicates a relative friction delta: $\Delta F = F(0) - F(V_{bow}L/x)$. *(All curves are smoothed.)*

This implies that the spectrum *envelope* of the "input signal", v_x (velocity of the string at bowing point), remains nearly constant as long as the slipping period remains constant, while harmonic frequencies are being shifted downward inside of the same envelope as a result of a (moderately) lowered fundamental.

Provided the rude simplification that v_x describes perfect square waves after slipping periods of T_0x/L are obtained, their spectrum envelope takes the function Y = | sin S | /S, where Y signifies the normalized amplitude, and $S = \pi / t_{SLIP}$. Hence, the frequencies n/t_{SLIP} (n=1,2,3...) cannot exist as part of the input signal, consequently neither as part of the output signal at the bridge.

However, due to the special characteristics of the transfer function between the string at bowing point and the bridge, the sinusoidal features of such an input spectrum appear not to be very recognizable when studying the spectrum of the output signal at the bridge (see figures 2 a and b):

The simulation model used for these figures included a bridge with "Cremer reflection"[4] (spring + dashpot, with a high delta function), nut reflection involving convolution with a wide gaussian function[1 5], and string torsion with low-pass reflection functions at both ends. Furthermore, the velocity of a non slipping bow was programmed to describe perfect square waves with positive periods of 2(L-x)/C and negative fly back (quasi slipping) periods lasting 2x/C, where C = transverse wave velocity, and x=L/7.

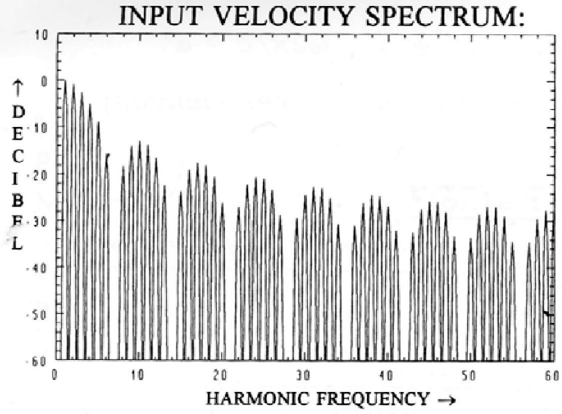

Figure 2a.

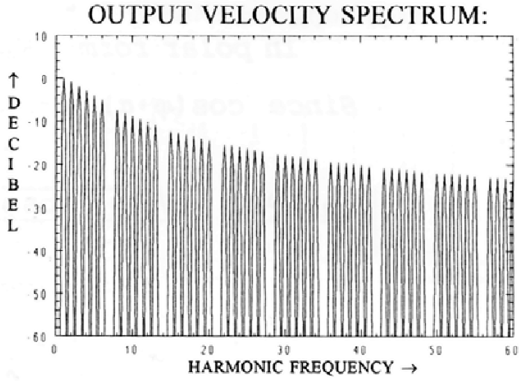

Figure 2b.

Transfer function string/bridge

The transfer function of an ideal string driving a bridge from a point of excitation located at a distance x from the bridge, can be developed in the following way (imagining zero reflection at the nut):

Z = *characteristic wave resistance of the string.*
$Z_{BR}(j\omega)$ = *impedance of the unstrung bridge* ($\omega = 2\pi f$).
$v_{BR}(j\omega)$ = *velocity of bridge.*
$v_X(j\omega)$ = *transverse velocity of the string at x.*
x = *distance from bridge to point of excitation.*
$F(x, j\omega)$ = *the exciting force.*
C = *transverse wave velocity.*
$T_{BR}(j\omega)$ = *velocity transmission at bridge.*
$R_{BR}(j\omega)$ = *velocity reflection at bridge.*

$$v_X(j\omega) = \frac{F(x, j\omega)\,[1 + R_{BR}(j\omega)\,e^{-j\omega 2X/C}]}{2Z} \qquad (1)$$

$$T_{BR}(j\omega) = \frac{2Z}{Z + Z_{BR}(j\omega)} = R_{BR}(j\omega) + 1 \qquad (2)$$

$$v_{BR}(j\omega) = \frac{F(x, j\omega)\,T_{BR}(j\omega)\,e^{-j\omega X/C}}{2Z}$$

$$= \frac{F(x, j\omega)\,[R_{BR}(j\omega) + 1]\,e^{-j\omega X/C}}{2Z} \qquad (3)$$

$$\frac{v_{BR}(j\omega)}{v_X(j\omega)} = \frac{[R_{BR}(j\omega) + 1]\,e^{-j\omega X/C}}{1 + R_{BR}(j\omega)\,e^{-j\omega 2X/C}} \qquad (4)$$

In polar form: $R_{BR}(j\omega) = (r,\varphi)$; $-\omega 2x/c = \alpha$.

Since $\cos(\varphi+\alpha) = -1$ *at the (impedance) resonances*:

$$\left|\frac{v_{BR}(j\omega_0)}{v_x(j\omega_0)}\right| = \frac{\sqrt{(1+r\cos\varphi)^2+(r\sin\varphi)^2}}{1-r}. \qquad (5)$$

In general:

$$\left|\frac{v_{BR}(j\omega)}{v_X(j\omega)}\right| = \sqrt{\frac{(1+r\cos\varphi)^2+(r\sin\varphi)^2}{[1+r\cos(\varphi+\alpha)]^2+[r\sin(\varphi+\alpha)]^2}}. \qquad (6)$$

If $\cos\varphi \neq -1$ *at* ω_0 $\rightarrow$ $\left|\frac{v_{BR}(j\omega_0)}{v_X(j\omega_0)}\right| > 1$; *and*

if $\frac{d\varphi}{d\omega} \neq 0$ *at* ω_0 $\rightarrow$ $\left|\frac{v_{BR}(j\omega_r)}{v_X(j\omega_r)}\right| \geq \left|\frac{v_{BR}(j\omega_0)}{v_X(j\omega_0)}\right|$

at a *maximum response* *frequency* $(\omega_r \approx \omega_0)$, *where* $\frac{d\left|\frac{v_{BR}(j\omega)}{v_X(j\omega)}\right|}{d\omega} = 0$.

The most interesting feature of this equation (see eq. 6) is the key role that the *phase* of the reflection at the bridge is playing: very high values of $|v_{BR}(j\omega)/v_X(j\omega)|$ can be reached if $\cos\varphi$ is unequal to -1, and/or its derivative with respect to frequency is different from zero.

It should be noticed that the transfer function above is independent of any reflection from the nut or the bow, thus independent of their respective admittances. However, the *impedance* of the string at point x is indeed a function of both bridge and nut reflections:

$$Z(x,j\omega) = 2Z\left[\frac{1}{1+R_{BR}(j\omega)\ e^{-j\omega 2X/C}} + \frac{1}{1+R_{NUT}(j\omega)\ e^{-j\omega 2(L-X)/C}} - 1\right] \qquad (7)$$

where $R_{NUT}(j\omega)$ = *velocity reflection at the nut.*

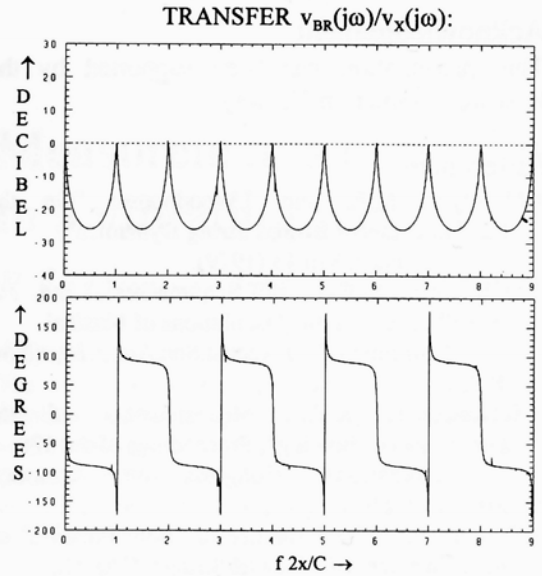

Figure 3: Transfer function with a purely resistive bridge.

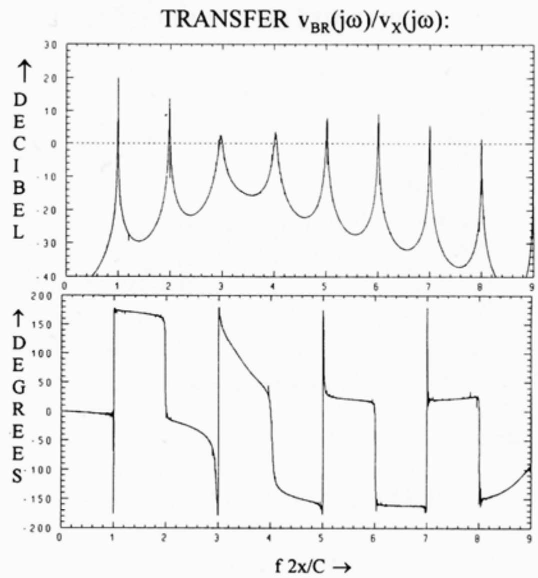

Figure 4: Transfer function with a resonant bridge.

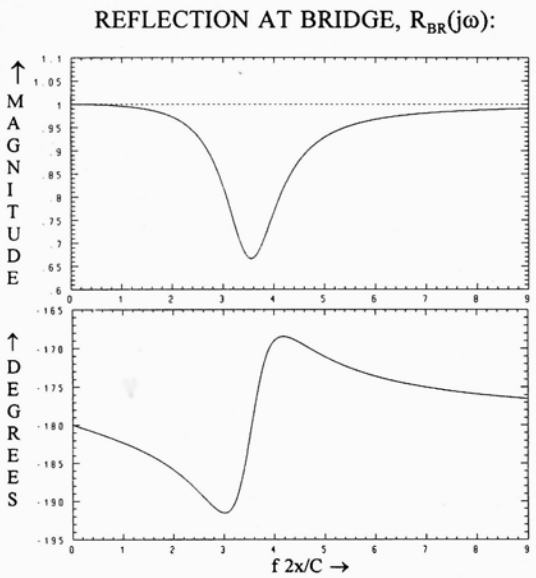

Figure 5: Reflection function of the bridge model utilized for the simulation of figure 4.

Figures 3 and 4 show the simulated transfer functions of two different bridge models. Figure 3: a purely resistive bridge, where the resistance of the unstrung bridge is 20 times greater than the characteristic wave resistance of the string. Figure 4: a resonant spring + mass + resistance model, to which the belonging reflection function is shown in figure 5.

These simulations were prepared by programming random (quasi white noise) bow velocities over 8192 time steps, after which a straightforward calculation of the FFT of v_{BR} with Hanning divided by FFT of v_X with Hanning was performed.

In the case of a purely resistive bridge, the transfer function takes maxima equal to unity and minima equal to Z/Z_{BR}. With the resonant bridge model of fig. 4 and 5, maxima up to +20 dB are noticed.

Transference with pitch flattening

In figures 2 a and b, the "node harmonics" (7., 14., 21., etc) were missing due to the integer ratio (7) between the quasi slipping- and the whole period. In figures 6 a and b, the same string model is used once more: this time it is bowed with constant velocity and a high "bow pressure", forcing a 14 cent pitch flattening on to the system. Now, the "node harmonics" appear at the output with considerable power in spite of the modest values these are holding in the input velocity spectrum (see figures 6 a and b):

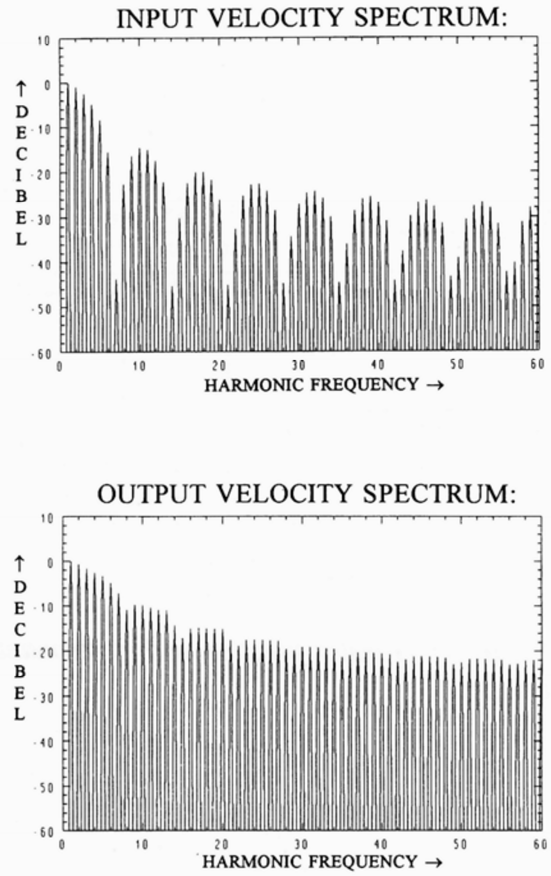

Figure 6: Spectra at bowing point (a) and the bridge (b) when a pitch flattening of 14 cent is simulated. Bow position is x = L/7.

Sound example

In order to exemplify the effects described above, the simulated velocity of a bridge was recorded during a series of five different bowing patterns: at all times the velocity of a non slipping bow was describing square waves with a "fly-back" period equal to T_0x/L. Pitch flattening was obtained by prolonging the positive (quasi static) period successively by $T_0/252$, thus flattening the pitch about 7 cent each time.

The initial whole period was $T_0 = 2L/C$. The simulations were performed with a compliant bow and a Cremer model bridge. Upon transference to audio, the high frequencies were boosted somewhat in order to emphasize differences in the overtone patterns. The series is played three times.

Acknowledgement

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References

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