The bowed string computer simulated —some characteristic features of the attack—

Knut Guettler

The Norwegian State Academy of Music P.B. 5190, Majorstua 0302 OSLO 3, NORWAY

1. INTRODUCTION

In this work computer simulation is used to study parameters influencing the starting transient of the bowed string. While many informative analyses of the bowed string during a steady state of oscillation have been published during the last few years, the conditions for the attack, which are indeed of great interest to the performer, have not been treated in detail. Of the many questions that arise, the following will be discussed:

—What are the requirements for reaching Helmholtz motion [1] as quickly as possible?

—How does string torsion interact with transverse motion?

—What is the effect of frictional losses in the string and the bridge with respect to playability?

—In which way can the characteristics of a bow influence its playing quality?

A systematic study of all parameters influencing the attack is not possible within a short paper. But simulated examples will demonstrate the basic influence of some important parameters on the initial transient of a bowed string.

2. THE COMPUTER PROGRAM "FIDDLE"

FIDDLE is a computer program based on the D'Alembert solution to the wave equation in the time domain. The two waves travelling in opposing directions along the string are expressed:

$$2.1)\quad n(x,t) = n_{+}(x-Ct) + n_{-}(x+Ct)$$

where n(x,t) is the transverse displacement of the string as function of the position (x) on the string, and the time (t). C is the wave velocity.

Input signal ("the stroke") is the bow pressure (z-force) and bow the velocity as functions of time, exciting the string at one programmable (zero width) position. The string is considered free of any loss in the transverse plane except at bridge and nut, where resistances and springs are individually programmed through first order differential equations. Torsional waves are treated similarly: they are reflected, but usually given large losses at high frequencies. Transverse and torsional wave impedances of the string are individually programmable as is the ratio of wave velocities. The frictional characteristic of the resin is given by a function of relative velocity in order to match curves obtained by Lazarus [2] and others. FIDDLE can also model bow compliance. Observations of its effect on gripping ability are discussed in this paper. The outputs demonstrated in this paper are the frictional force and the displacement of the string at the bowing point. The frictional force is calculated as

$$2.2)\quad f(t) = \frac{2Z_{TRV}Z_{TOR}}{Z_{TRV}+Z_{TOR}}\,\Delta V(t)$$

where Z_{TRV} = transverse wave impedance (characteristic resistance) of the string;
Z_{TOR} = torsional wave impedance; $\Delta V(t)$ is the difference between the calculated string surface velocity at the bowing point and the velocity of the same surface assuming that friction instantly dropped out at the beginning of the time-step in question.

3. SIMPLIFIED ANALYSIS OF AN ATTACK

In order quickly to create a Helmholtz movement in the string, it is crucial to obtain only one slip and one grip during each period of the fundamental frequency *as early as possible.* This may be obtained already during the first period, but depends on a delicate balance between bow pressure and velocity.

The frictional force curve of a "perfect" attack may look as illustrated in Figure 1:

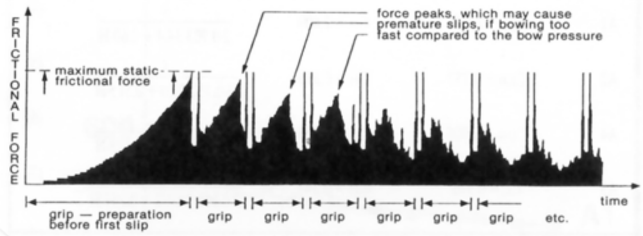

Figure 1: Frictional force during a successful attack (simulated).

Peaks in force appearing during the periods of static friction will cause premature slips if they exceed the maximum friction. While static frictional force is a function of the bow velocity during the transient (see Eq. 3.1), the maximum friction is determined by bow "pressure".

For the torsionless, perfectly flexible string with no losses and total reflection at each end, the **static** frictional force can be expressed by the following equation:

$$3.1)\quad \mathbf{f(t)} = \mathbf{2Z_{TRV}}\left[\mathbf{V_B(t)} + \sum_{i=1}^{\infty}\mathbf{V_Y}\,(t-it_1) + \sum_{j=1}^{\infty}\mathbf{V_Y}\,(t-jt_2)\right]$$

where Z_{TRV} = transverse wave impedance (real quantity); $V_B(t)$ = bow velocity as function of time; V_Y = velocity of the string at the point of excitation (X); L = length of the string; $0<X<L$; $t_1 = 2X/C_{TRV}$; $t_2 = 2(L-X)/C_{TRV}$; C_{TRV} = transverse wave velocity. V_B and V_Y are both zero for $t<0$.

When the friction is **sliding**, the velocity of the string at X is:

$$3.2)\quad V_Y(t) = \frac{f_{sl}}{2Z_{TRV}} - \sum_{i=1}^{\infty} V_Y(t-it_1) - \sum_{j=1}^{\infty} V_Y(t-jt_2)$$

where f_{sl} = sliding friction.

The simulation of such a string bowed with constant velocity and pressure is demonstrated in Figure 2. The slips are synchronous with the fundamental frequency until a double period occurs after the 6th period. (X = L/8.) In general: after (L−X)/X periods, the subsequent release will be cancelled due to interference between the frequencies f_0, $1/t_1$ and $1/t_2$. An increase in bow velocity is indeed required if the synchronization of release is to be maintained - as pointed out by L.Cremer [3,4].

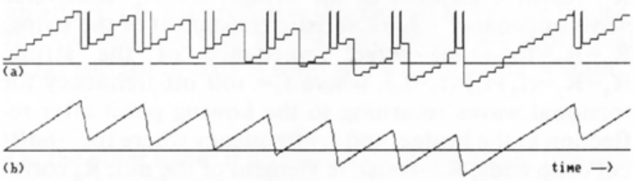

Figure 2: Flexible string without losses. (a): frictional force during an attack with constant bow velocity and pressure. Broken X-axis indicates sliding friction. (b): displacement of string at bowing point.

With this system the requirements for obtaining a first period composed of "one slip/one grip" when bowing with constant, positive bow velocity and pressure are:

$$3.3)\quad f(t_{rel-} + t_2) < f_{max} \text{ and}$$
$$f(t_{rel+} + T) > f_{max}$$

when using equation (3.1) as for static friction. T = period of the fundamental frequency; t_{rel} = time of the first release.

This leaves only a small range of acceptable starting velocities: $V_B$min through $V_B$max.

Detail from comparison of two simulations (with losses) is demonstrated in Figure 3: X=L/8. $V_B$max is only 17% greater than $V_B$min. Typically, $V_B$max/$V_B$min is near to L/(L−X). The tolerance of V_B is caused by the drop in frictional force starting at the time t_2 after the first flyback of the string.

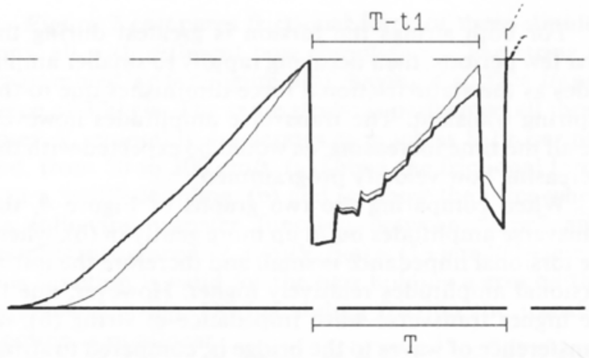

Figure 3: Frictional force between bow and string. Two different bow velocities are simulated, both allowing for one slip and one grip during the initial period. Fine line: close to $V_B$max. Bold line: close to $V_B$min. Maximum frictional force is indicated by the horizontal, dotted line.

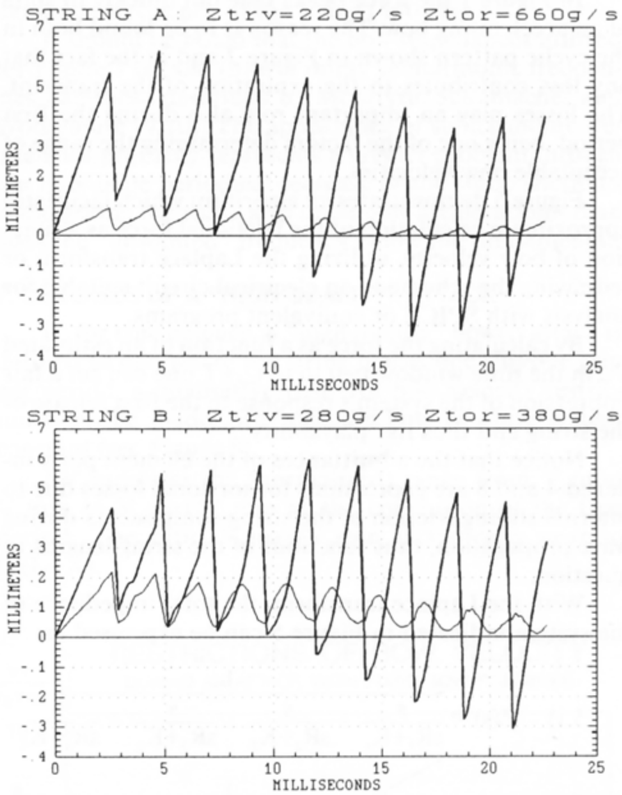

Figure 4: Strings of different impedance characteristics. Transverse (bold line) and torsional (fine line) displacements of the string at bowing point with respect to equilibrium during a simulated attack. V_B increasing from 24 to 42 cm/s.

4. TORSIONAL WAVES INCLUDED

Figure 4 (a) and (b) compare the attack transients of two strings with different wave impedance ratios: (a) Z_{TRV} = 220 g/s and Z_{TOR} = 660 g/s; (b) Z_{TRV} = 280 g/s and Z_{TOR} = 380 g/s. These impedances are within the range found in the violin D-strings tested by Schumacher [5] and Pickering [6]. Because the mechanical series impedances of the two strings (165 g/s and 161 g/s respectively) fall very close together, both can be treated with the same stroke during the attack. The strings are tuned to 440 Hz; bow pressure is 1 N.; L/X=6; V_B starts at 24 cm/s increasing 220 dB per second (5.9% per period); C_{TOR}/C_{TRV} = 6.0 and 3.0 respectively. Reasonable losses are introduced at bridge, nut and for the reflection of torsional waves. Torsional displacement is to be understood as radians times radius.

For both strings the torsion is greatest during the first few periods, then decaying rapidly to smaller amplitudes as the static frictional force diminishes due to the expiring transient. The transverse amplitudes however, are all the time increasing, as would be expected with the increasing bow velocity programmed.

When comparing the two graphs of Figure 4, the transverse amplitudes build up more gently in (b), where the torsional impedance is small and therefore the initial torsional amplitudes relatively higher. However, due to the higher transverse wave impedance of string (b), its transference of waves to the bridge is, compared to string (a), the greater most of the time. Spectrum analyses of their two transients reveal significant differences, but this complex matter has yet to be interpreted mathematically.

5. THE EFFECT OF LOSSES

In Figure 1 the force peaks fade out quickly in spite of an accelerating bow. The reason is to be found both in the cyclic pattern shown in Figure 2 and in the fact that any loss contributes to the expiration of the transient. The losses play an important role also during the first period, being one of the factors determining the range of acceptable bow velocities.

Figure 5 demonstrates an algorithm which facilitates approximate calculation of the frictional force as a function of bow velocity, utilizing the Laplace transform or redrawing the scheme to an electrical circuit suitable for analysis with SPICE or equivalent programs.

By calculating the force as a function of an estimated V_Y in the time window $t=0$ to $t=t_{rel}+T$ one can get a fair impression of the system's response to the first release of the string and thus its "playability".

Notice that the admittances of the element pairs indexed 4 and 8 are expressions for torsional losses due to internal sliding friction of the string components during wave propagation, thus functions of the string lengths in question.

With the Laplace transform, the total impedance of the system indicated in Figure 5 can be expressed as:

$$5.1)\quad Z(s) = \frac{1}{\frac{s}{sR_1+K_1}+\frac{s}{sR_2+K_2}+\frac{s}{sR_3+K_3}+\frac{s}{sR_4+K_4}} + \frac{1}{\frac{s}{sR_5+K_5}+\frac{s}{sR_6+K_6}+\frac{s}{sR_7+K_7}+\frac{s}{sR_8+K_8}}$$

The force between the bow and the string is then found as

$$5.2)\quad f(s) = V_Y(s)Z(s)$$

Transformed to the time domain, the force will, with a constant (string surface) velocity starting at t=0, give the following equation:

$$5.3)\quad f(t) = M + Nt + Oe^{-ot} + Pe^{-pt} + Qe^{-qt} + Ue^{-ut} + Ve^{-vt} + We^{-wt}$$

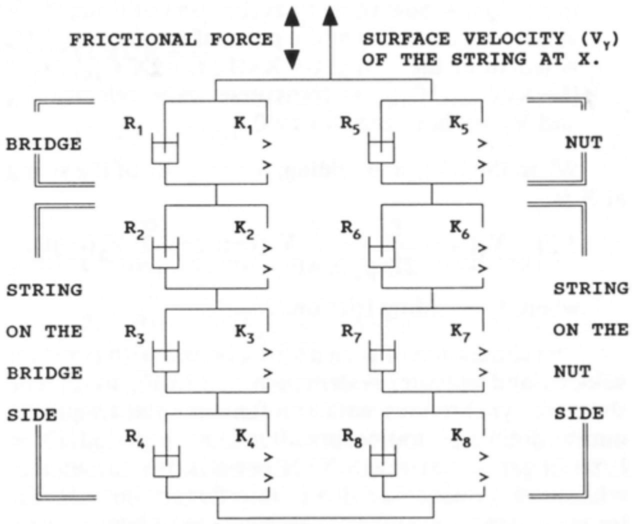

Figure 5: Circuit allowing for Laplace calculation of frictional force during the first period of an attack: R_1=resistive element of the bridge; $R_2=R_6$=transverse wave impedance (characteristic resistance) of the string; $R_3=R_7$=torsional wave impedance of the string; $R_4=R_3=(f_2+f_1)/(f_2-f_1)$, where f_1= roll off frequency for torsional waves returning to the bowing point after reflection at the bridge, and f_2=frequency where the −6dB/oct drop ends; R_5=resistive element of the nut; R_8 corresponds with R_4 on the nut side of the bow.

K_1=spring element of the bridge; $K_2=2R_1/t_1$; $K_3=2R_3/t_3$, where $t_3=2X/C_{TOR}$, and C_{TOR}=torsional wave velocity; $K_4=2\pi f_1(R_3+R_4)$; K_5=spring element of the nut; $K_6=2R_5/t_2$; $K_7=2R_7/t_4$, where $t_4=2(L-X)/C_{TOR}$; $K_8 2\pi f_3(R_7+R_8)$, where f_3 corresponds with f_1 on the nut side.

Figure 6 (a) and (b) compare Laplace calculations (fine, smooth lines) with the frictional force curves of two attacks simulated with bowing point near the bridge (X=L/16). The maximum frictional force is indicated by horizontal, dotted lines, while the vertical dotted lines indicate changes between sliding and static friction. Bow velocity, pressure and all other parameters are equal in (a) and (b), with exception of R_1, which in (a) has been chosen to simulate a great frictional loss at the bridge; in (b) this loss is significantly smaller.

V_B has been chosen to facilitate a perfect attack with the system of (a), but, as can be seen from (b), the same bow velocity is too small for a system with a better reflecting bridge, resulting in a prolonged first period. Notice how the frictional force builds up differently after the slipping period in the two cases.

In general: substantial loss of energy across R_1 and/or R_4 - or any similar element introduced on the bridge side - demands a reduction of bow velocity during the attack, no matter where the energy loss is situated.

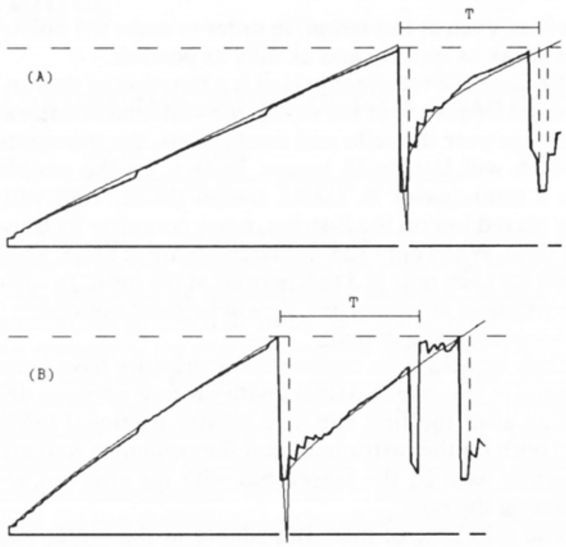

Figure 6: Frictional force: simulation (bold line) compared to Laplace-calculation (fine line) of the first part of an attack. (a): high resistive (frictional) loss at the bridge. (b): low resistive loss. Bowing parameters are kept equal in (a) and (b). X-axis is solid when the friction is static.

On the other hand: if no loss exists of the higher frequencies in the reflection of torsional waves, premature slips are almost certain to take place due to high rising spikes in the static frictional force.

6. THE EFFECT OF BOW COMPLIANCE

In all the examples referred to above, the bow has been "stiff", meaning $V_B = V_Y$ whenever the friction is static. A violin bow has some compliance in its longitudinal direction, so it would be more realistic to consider V_B as merely defining "an inertial frame" for the velocity of the bow hair touching the string (V_H) (Schumacher [7]). If the simulated bow is given a flexible coupling, i.e. a frequency-dependent transmission between V_B and V_H, it can be shown that transients [8] fade out much more rapidly: the bow introduces damping of all waves in the string.

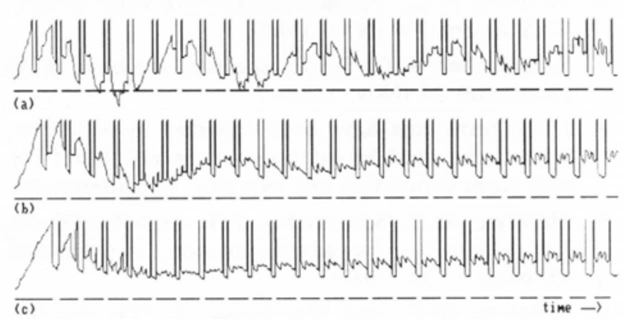

Figure 7: Frictional force during an attack with the bow accelerating 8.2% per period. (a): no compliance in the bow; (b): 3.3 dB loss in the bow/string transmission above f_0; (c): 3.3 dB loss above $0.5f_0$.

Figure 7 compares frictional force of three simulations, all with different bow compliances. The string is programmed as for Figure 4(a), having a surface impedance of 330 g/s. The same stroke is performed all three times: V_B smoothly accelerates as much as 8.2% per period, from 50 to 300 cm/s. (a) shows the frictional force with a "stiff" coupling, (b) and (c) show the force when the following transfer function between "bow" and "hair" is introduced: $V_H(j\omega)/V_B(j\omega) = (1+j\omega\tau_2)/(1+j\omega\tau_1)$. This function is valid for the hair holding a grip on the unreflected string. During sliding friction the transfer function is nonlinear.

For (b) the (mechanical parallel) impedance of the bow is 0.707 Kg/s + 1,955 Kg/s_2, which gives $\tau_1 = 1/2\pi 300 s^{-1}$, and $\tau_2 = 1/2\pi 440 s^{-1}$ during static friction. For (c) the impedance is 0.707 Kg/s + 977 Kg/s^2, which gives $\tau_1 = 1/2\pi 150 s^{-1}$, and $\tau_2 = 1/2\pi 220 s^{-1}$. As can be seen, the shortest transient occurs when the (−3.3 dB) loss in the transmission ranges from well below the fundamental frequency of the string (440 Hz).

It should be noted that the bow impedances utilized for Figure 7 (a) and (b) will imply subharmonic frequencies like those described by McIntyre, Schumacher and Woodhouse [9,10]. The appearance of subharmonic frequencies probably represents a greater problem through the inevitable periodical increase of the static friction - endangering the static grip upon bow acceleration - than does the "unwanted" frequency below the fundamental.

7. OUTLINES OF A "HIGH QUALITY" ATTACK

If, for convenience, a "high quality" bowed attack is defined as one where the string makes one slip/one grip per period, a qualitative figure of bow velocity restrictions can be made, provided the bow pressure is kept constant (see Figure 8).

ΔV indicates the (normalized) range from $V_B$min/$V_B$min to $V_B$max/$V_B$min at the conclusion of the first period. The ratio $V_B$max/$V_B$min is typically near to L/(L−X).

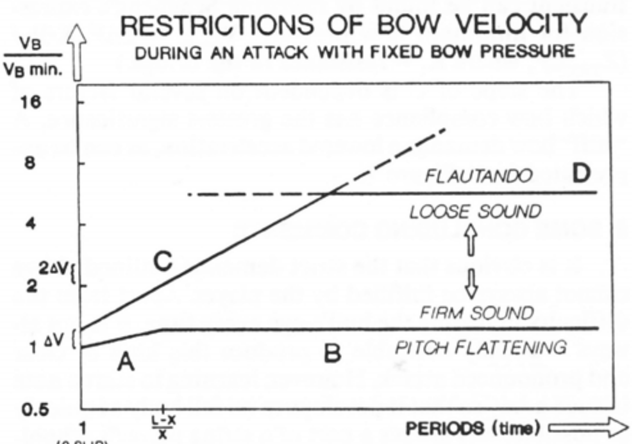

Figure 8: During an attack the bow velocity should follow a path inside the frame of A through D in order to create Helmholtz motion as quickly as possible.

Below ΔV prolonged periods will occur between each slip. Provided an accelerating bow, the string will typically synchronize to the correct frequency as soon as the range ΔV is entered. In practical playing, this lower range must be regarded as desirable too, as long as the bow velocity reaches ΔV within not too many periods.

$2\Delta V$ indicates a range where the fundamental frequency will be greatly suppressed, while the 2. harmonic will take over as the dominant fundamental.

Between the two ranges, ΔV and $2\Delta V$, several variants may occur, all including at least two slips per period.

$V_B$min and $V_B$max will be lowered if:

—the ratio X/L is lowered

—the combined wave impedance, $Z_{TOR}Z_{TRV}/(Z_{TOR}+Z_{TRV})$, is increased

—the difference between max. static and sliding frictional force is lowered

—losses on the bridge side of the bowing point are increased.

V_B must typically be accelerated about 15 - 40% during the (L−X)/X first periods in order to avoid any prolonged sticking (see line A in Figure 8), after which time it may remain constant (see lower limit, line B), or, in some cases even retard to a value slightly below the initial $V_B$min. (If one ignores torsion and spring elements of the system, the minimum V_B after the transient can be found by rewriting Schelleng's expression for maximum bow pressure [11]:

$V_{Bmin'} = \Delta F_Y X/(Z_{TRV}L)$, where ΔF_Y = maximum static friction minus sliding friction.)

Curve C describes the greatest possible velocity gain - where V_B is increasing *exponentially* from the initial $V_B$max. With the X-lin/Y-log coordinates of Figure 8, C can be drawn as a straight line until it smoothly bends to horizontal and joins D. D describes the *maximum after transient bow velocity* for the given bow pressure. This velocity value will be lowered if X/L or ΔF_Y is lowered, or the losses or Z_{TRV} increased. (Once more ignoring torsion and spring elements of the system, the maximum V_B after transient can be found by rewriting Schelleng's expression for minimum bow pressure: $V_{Bmax'} = 2\Delta F_Y X^2 R_1/(Z_{TRV}L)^2$, where R_1 = resistance of the bridge.)

The slope of C is dependent on several factors of which bow compliance has the greatest significance. A "stiff" bow demands a lowered acceleration, as can be appreciated from Figure 7.

8. SOME CONCLUDING COMMENTS

It is obvious that the strict demands outlined above cannot always be fulfilled by the player. Apart from the difficulty in hitting the bull's-eye every time, it is not always musically desirable to produce this kind of clear and pronounced attack. However, learning to start a note in such a way so that it develops to its full body as quickly as possible, was always a part of a string player's schooling. The closer the hit is to ΔV, the faster sonority appears as well as the possibility to chose the most desirable tone color. The skilled player will treat notes played in different positions or on different strings with individual bow velocities, even at fast tempi, in order to make the instrument speak as quickly and as fully as possible.

Because the transient period is a function of the fundamental frequency of the string, the violin has a natural advantage over the cello and double bass, the transients of which will last much longer. In fact: on the double bass, a semi-quaver in march tempo (M.M. 120) will, when played low on the E-string, never complete its transient time. When only 125 milliseconds are at hand, what you get for each note is 3 to 6 periods at the most. In such circumstances any incorrect V_B will be most noticeable - and sonority just a dream!

Only attacks with constant bow pressure have been analyzed in this paper. Attacks with the bow pressure decreasing after the first slip give greater frictional tolerances both on the instrument and the computer, and are frequently used by the player, but with the effect of accentuating the note.

The influence of mass impedance at the bridge has only been investigated to a limited degree by the author, but a mass obviously affects the static frictional force during the attack when playing near the bridge, thus also affecting the tolerances of the bow velocity.

REFERENCES

1. Helmholtz, H.V., "Lehre von den tonempfindungen" [On the sensation of tone] (1862), Braunsweig: Vieweg.
2. Lazarus, H., "Die behandlung der selbsterregten kippschwingungen der gesrtichenen saite mit hilfe der endlichen Laplacetransformationen" Dissertation, Der Technischen Universität Berlin (1972).
3. Cremer, L., *The Physics of the Violin* Chapter 8, pages 184-188. The MIT press (1984).
4. Cremer, L., "Considerations of the duration of transients in bowed instruments" CAS Newsletter #38 (Nov. 1982).
5. Schumacher, R.T., "Measurements of the parameters of bowing," Proceedings of the 12th International Congress on Acoustics, Toronto, 1986.
6. Pickering, N.C., "Physical properties of violin strings," CAS Journal 44 (Nov. 1985).
7. Schumacher, R.T., "Some aspects of the bow," CAS Newsletter 24 (Nov. 1975).
8. These transients have been reported by McIntyre, Schumacher and Woodhouse in the article ref [9], both from simulation and practical measurements.
9. McIntyre, M.E., Schumacher, R.T., and Woodhouse, J.: Aperiodicity in bowed-string motion," Acustica Vol. 49, pages 13-32, (1981).
10. McIntyre, M.E., and Woodhouse, J.: "On the fundamentals of bowed-string dynamics," Acustica Vol. 43, pages 93-108, (1979).
11. Schelleng, J.C., "Pressure on the bowed string," CAS Newsletter #13 (May 1970).