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ON SIMULATION AND ANALYSIS OF THE BOWED STRING - TECHNIQUE AND TOOLS

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ABSTRACT

Many methods are available for simulation and analysis of the bowed string. This paper discusses some tools that have proven handy when the bowed string is subjected to numeric analyses. The author's simulation algorithm is based on the McIntyre-Woodhouse-Schumacher model, which is constructed on D'Alembert's wave equation combined with convolution functions at the reflecting string terminations. Each of these functions shows specific influence on the system's playability. By introducing the bow's impulse response as part of a convolution integral applied in connection with the frictional force, valuable information can be obtained on how the bow characteristics may affect the sound output.

Because the steady-state oscillation of a bowed string shows no inharmonicity, utilization of L. I. Bluestein's FFT algorithm (which accepts any arbitrary number of elements) gives advantages for spectrum comparison. Also, the identification of the systems point impedances and transfer functions can efficiently be performed with the aid of a windowed "chirp" function. The realism of the hyperbolic friction model is discussed in the light of J. Smith's dissertation on frictional laws. Realistic bow-stroke functions are suggested for easy parameter input. Presentation of the computed results can take several forms: both moving graphics and audio provide valuable information besides the traditional graphs and tables.

ON THE REFLECTION FUNCTIONS

Any wave excited on a string will be reflected at the string terminations and/or at any object causing a change of impedance in the transmission link. The nature of these reflections plays a crucial role in defining the "playability" of the system. In his article "Der Einfluss des 'Bogendrucks' auf die selbsterregteren Schwingungen der gestrichte Seite"[1], L. Cremer describes two reflection models which both (to the author's understanding) are meant to account for reflections at the string's end as well as dissipation along the string. The first one is expressed: "*...we multiply the displacement of the Helmholtz motion by a damping term* $exp(-n\delta_1 t)$ *where n is the number of harmonic and* δ_1 *the decay rate of the fundamental..*". The second one is described as the combination of dashpot and spring, thus giving a reflection with

"..*an asymptotic constant value at high frequencies*".

There are several very significant differences between the two: the first one implies no dirac delta, while the second one indeed does when using the figures suggested by Cremer. Only the first expression is capable of producing the effect of "pitch flattening" (lowered fundamental frequency due to excess bow "pressure"), while the second expression will be insensitive to differences in the bow force, nearly until the periodic triggering breaks down. The first one ensures damping of high mode frequencies while the Q-values of the second system are rapidly increasing with frequency (see Woodhouse[2]).

Dispersion can be accounted for by choosing skewed functions, but most common so far has been to convolute with a gaussian occupying some 3-6% of the nominal first-mode period by the nut and less by the bridge. One problem here is that the nut-side gaussian does not provide sufficient damping of the lowest modes. One way around this might be to reduce the area under this function from unity to, say, 0.98 and to even less for reflection of torsion. Otherwise, the combination of convolution with a unity-valued gaussian and multiplication with a dirac delta holding the same reduced value (e. g., 0.98), followed by an exponentially decaying tail (the dashpot-spring model) gives good control combined with computational efficiency. A small price is then paid in form of a slight skewness of the function.

When deciding for reflection functions for the bridge and nut, one should evaluate their effects on the system's Q-values separately: the losses' influence on the model's playability depends as much on *where* they are situated as on their values. For instance: on the bridge side, high admittance in the upper frequency range tends to increase playability, while on the nut side, the same feature makes the system more exposed to pitch flattening.

Resonant functions

So far the general damping of the system has been the main issue. However, the convolution technique provides the option to program any impulsive response for end reflections as well as for the bow. These may be based on calculations or actual registrations on real instruments and bows. (When computing with measured impulsive responses for the bridge or nut, these should of course be convoluted with some loss functions modeling losses of the string length in question, before implementation in the model.) An example of the resonant bridge is given by McIntyre and Woodhouse in an article[3] where the "wolf note" is discussed.

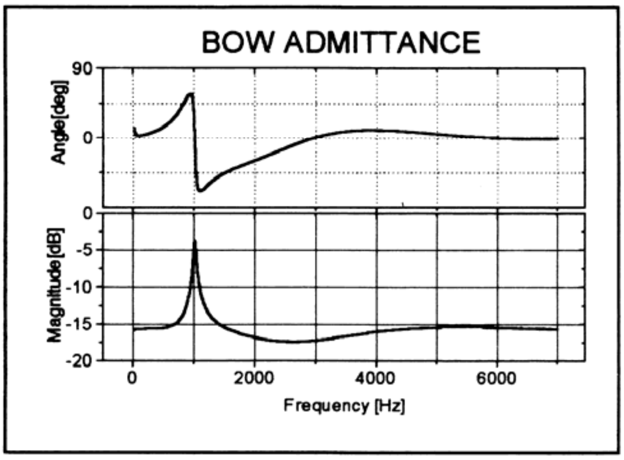


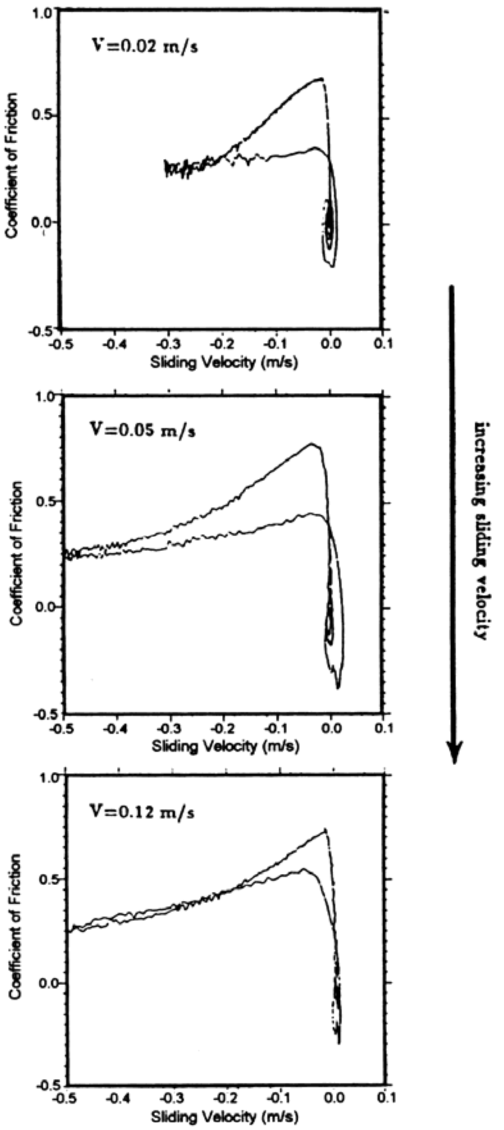
Figure 1:
Through convolution, the bow can be given resonances. If the admittance peak at 1 kHz is near to the transverse point admittance of one of the mode frequencies, the interaction will be noticed as a reduction of power in the output spectrum [4].

In the same way as for the string-end reflections, the bow admittance function may be programmed as a combination of an impulsive response to be convoluted, and a dirac delta with an exponentially fading tail. The dirac function makes it easy to adjust the minimum admittance to a given value. The bow admittance is principal in the damping of the onset transient.

(Measurements performed by A. Askenfelt suggest ca 0.01 s/kg as the bow hair's minimum longitudinal admittance - while peak values are found near 1 s/kg for some frequencies between 1500 and 2200 Hz [5].) However, because the bow's impulse response becomes part of a digital feedback loop similar to an IIR filter, care should be taken to avoid instability.

THE FRICTION FUNCTIONS

In his doctorate dissertation for the Technical University of Berlin (1972), H. Lazarus gives examples of friction characteristics from his own measurements[6]. A more recent study has been carried out by J. H. Smith in his doctorate dissertation on "Stick-slip vibration and its constitutive laws"[7]. Basically, the hyperbolic model remains valid, but Smith proves that the return path from sliding to static friction is a different one and shorter than from stick to slip (see Figures 2 a-c: the rotation is anti clockwise at the upper parts).



Figures 2 a - c:
Friction-velocity characteristics for a rosin coated steel rod and a perspex wedge, showing the effect of increasing the sliding speed (from J. H. Smith).

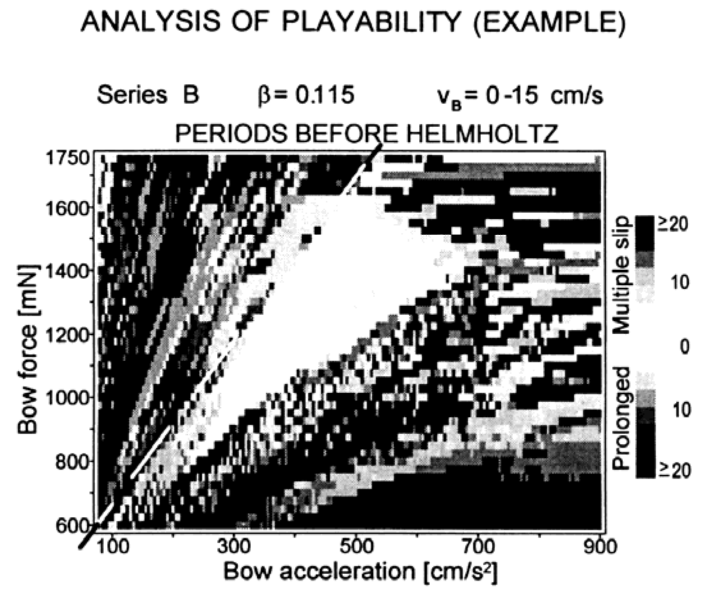
Mathematically, this is what was McIntyre, Woodhouse and Schumacher[8],[9] already did imply in their models, but Smith shows that this holds true even when the relative velocity between the two sliding materials is continuous, as it is between the bow's hair and a string with stiffness. The error originating from the idealization of the string is therefore less significant than one could have feared.

Smith also discusses time dependence of the limiting static frictional force. This has been included as an option in the present author's simulation model, but so far major effects of this feature have not been observed. One noticeable consequence, however, is a slight reduction in playability if the static limiting force has been increased during a long static onset preparation, and then suddenly drops after the first slip. This effect is quite observable with some modern (and quite sticky) double bass rosins.

THE DYNAMIC INPUT PARAMETERS ("THE STROKE")

Most of the stroke parameters (normal bow force and bow velocity) that have been utilized by researchers over the years start the stroke onset with two nonzero values (including many of those simulated by the present author). Although this is physically not realizable, it was tempting to do so when computing with older and slower machines, as the interval before "action" was minimized this way. However, at present, more realistic parameters should be chosen, particularly when investigating the playability of a system.

Figure 3:
The number of periods preceeding the Helmholtz triggering is an important indicator of percieved quality of the attack [10]. For the open G-string of a violin, the limit for acceptance of pre-Helmholtz transient has been found to be 90 ms (18-19 nominal periods) for onsets with multiple slips (i. e., all attacks shown below the drawn diagonal), for prolonged periods the limit was 50 ms (above the diagonal). The set consists of 7755 simulations, each with a fixed normal bow force and the bow accelerating from 0 to to a final value of 15 cm/s. Only the stick/slip information was saved during simulations. (Original graph in colors.)



At the stroke onset, the options are: (1) zero velocity/nonzero force - as when starting the bow on the string; (2) nonzero velocity/zero force - as in legato crossing to a new string; (3) both zero valued - is more rare. For a normal détaché (the most common stroke) option (1) should be used, with the bow acceleration to a final velocity while keepeing the bow force static or changing somewhat with time[11],[12]. The following functions are most useful when programming strokes, especially when covering a large parameter space such as shown above.

- *Linear acceleration (or force gain)* [initial value][final value][rate]
- *Exponential acceleration (or f. g.)* [initial value][final value][rate in dB/time unit]
- *From - to* [initial value][final value][starting time step #][ending time step #]

The following two are recommended for system identification:

- *Chirp* [amplitude][initial frequency][final frequency] [window] (see "Two useful tools")
- *Cosine/Sine* [amplitude][frequency]["DC component"]

All these functions should be combinable through addition, and give warning if the final value cannot be reached within the number of time steps programmed. The "rate" brings up the question whether to program for a dimensionless system (like McIntyre/Woodhouse/Schumacher) or use dimensions for time/velocity/force, etc. The dimensionless system is very appealing, with one exception: the curvature of the friction characteristics has to change with the dimensionless velocity, it needs recalculation at each time step as long as the derivative of the bow velocity is nonzero.

THE SIMULATION OUTPUTS

It is the author's observation that writing the computed output parameters to disk (even to a RAM drive) occupies more time than the actual computation of their values. Typically: for a model with 180 time steps per nominal period and gaussian functions of 5 and 3 elements by the nut and bridge respectively, a 486-PC of 50 MHz will compute some 11-12 hundred time steps per second if the writing to disk is omitted. (This describes a program written in FORTRAN/2 computing with double precision.) When writing six variables to file in *double precision* at the conclusion of each time step, the rate is lowered to a mere one hundred, while writing one *integer* variable only gives an output of some 800 time steps per second. For many analyses, the slip-stick pattern is the only information of interest (see Fig. 3). This can of course be expressed as zeroes and ones, occupying minimal time and space. Analyses based on force, velocity, etc., require considerably more time, especially when stored as nonintegers. Consequently, the selection of output parameters should at all times be reflecting the task in question. The outputs readily available to the author as standard routines of his program are:

- *slip/stick*
- *transverse displacement/velocity of the string at any point*
- *torsional displacement/velocity of the string at the point of bowing*
- *transverse position/velocity of the bridge*
- *transverse force on the bridge*
- *frictional force between the bow and the string surface*
- *transverse and torsional signals arriving at the bow after reflection by the nut/bridge*
- *a video file (matrix), which keeps a graphic record of the movements of the entire string*
- *all internal numbers used for the computation (to be used for a future continuation of the stroke)*

The force on the bridge may also be written in 16-bit format as *a sound file* after convolution with an appropriate transfer function. (It should be mentioned here that without the information on the four signals arriving at the bow, analyses like the one on "Anomalous Low Frequencies - ALF"[13] would be very hard to carry out. At this point, D'Alembert's wave equation gives some benefits.)

For the parameters above, the author has a number of built-in routines for analyses, some of which may be producing printouts in intervals during the simulations. A routine for determining the Q-values is paramount. In addition to the main input parameters changing with time (bow velocity and normal bow force) the string may be programmed to be initiated with any waveform propagating in each direction. This is particularly useful when estimating the impulse or pulse responses of different parts of the model.

TWO USEFUL TOOLS

The L. I. Bluestein linear filter

When the oscillation of a bowed string has reached a steady state, no inharmonicity exists. If using the normal Cooley-Tuckey FFT algorithm for spectral analyses, windowing or zero padding will be necessary in most cases. Within the 60 dB-band of interest, several important features may then be obscured, especially if the time window is short. The filter approach suggested by L. I. Bluestein[14] lets you choose an arbitrary number of elements in your window, while still utilizing the nth-power-of-two-element-based Cooley-Tuckey routine provided by most programs for analysis. In principle, the Bluestein filter convolutes the time-domain signal with a complex chirp, which impulse response is $h_r = exp(+j\pi r^2/N)$, where N is the number of elements wanted for the time window under analysis. After the convolution, which is executed as a multiplication of the Fourier-transformed signal and filter padded to a minimum of $2N$-1 elements to reach the nth power of two, the excess elements are simply removed, carrying no information of interest. Figures 4 (a) and (b) give a good example of the effect.

The chirp

In order to identify the system, it is necessary to feed it with an input signal that carries power in the entire frequency band of interest. Furthermore, the spectrum of the signal should be smooth with little or no differences in the power of adjacent frequencies. The chirp is a signal of this kind, which may be programmed as bow velocity in combination with a normal bow force high enough to ensure static grip at all times. The definition of the (discrete) chirp is

$$(1) \qquad x(n) = A \cos \phi(n), \qquad \text{where } \phi(n) = a + bn + cn^2.$$

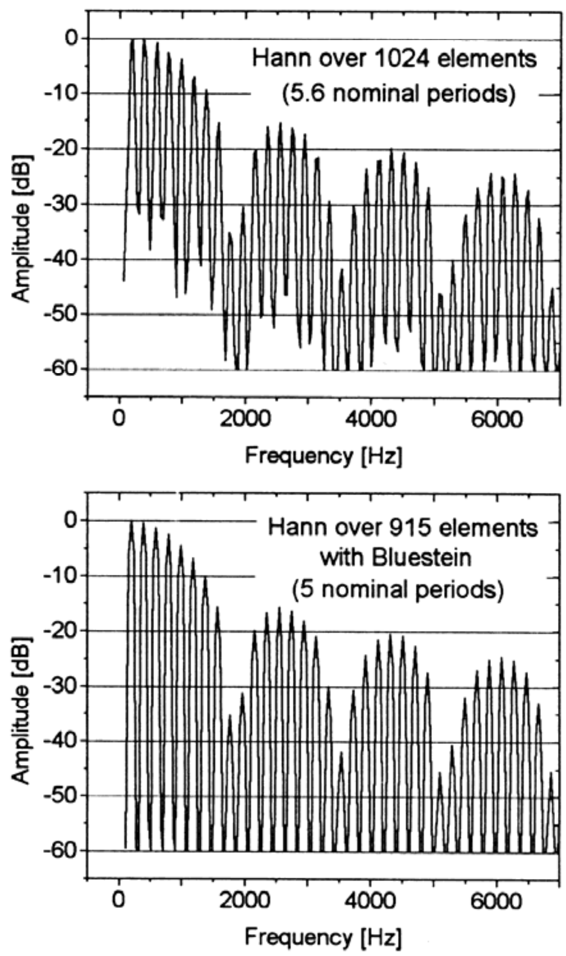
Within the frequency band $f_{HIGH} - f_{LOW}$, the angular function can be specified

$$(2) \qquad \phi(n) = 2\pi[nf_{LOW} + n^2 (f_{HIGH} - f_{LOW})/2N] + \alpha \ ,$$

where N is the size of the chirp and α is a constant. For normal system identification, α and f_{LOW} should both be set to zero, which reduces the expression to

$$(3) \qquad \phi(n) = 2\pi[\ n^2 f_{HIGH} /2N] \ .$$

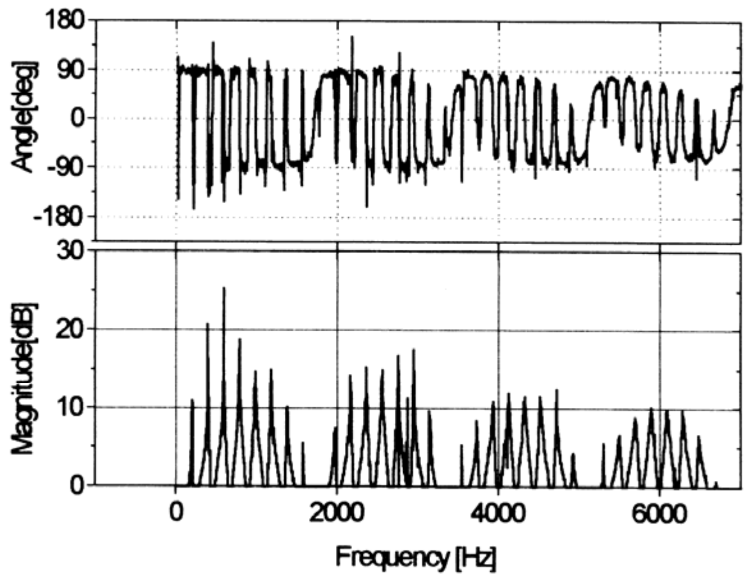
Because $x(n)$ (or rather $x(t)$) will have nonzero derivatives at the high frequency end, noticeable leakage will occur due to discontinuities if a smoothing function is not applied. It is therefore advisable to expand the frequency band some 10% and apply the second half of a Hann window on the second half of the function $x(n)$ to ensure a smooth fade out.



Figures 4 a and b:
A bowed string in steady-state oscillation shows no inharmonicity. Even during the transient the inharmonicity is small when a regularly paced triggering has started. It is often desirable to make FFTs over only a few numbers of nominal periods. The L. I. Bluestein filter provides tools for choosing the number of elements arbitrarely. The upper graphs shows the amplitude spectrum of string velocity at the bow (simulated), utilizing the Hann window over 1024 time steps, which comprises 5.6 nominal periods.

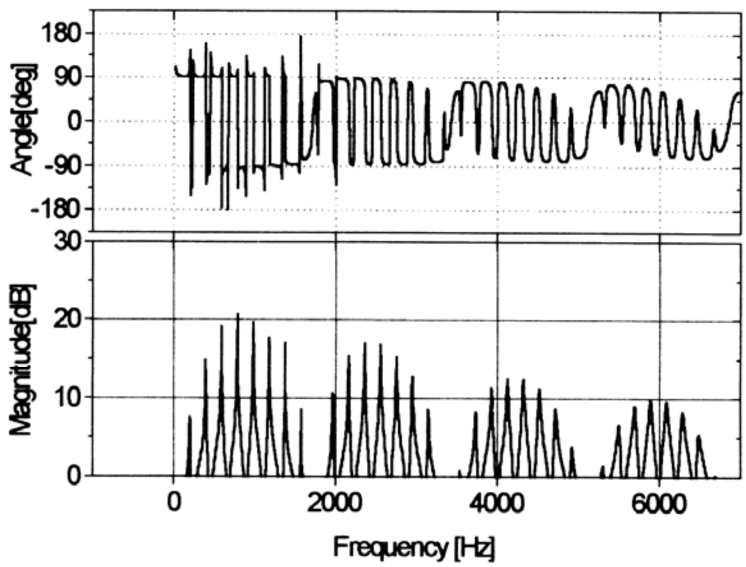
The lower graph shows the same signal with only 915 time steps included in the FFT. Deviations from steady state would have been visible in the lower part of this plot, but obscured with normal FFT routines.

TRANSVERSE STRING-POINT ADMITTANCE

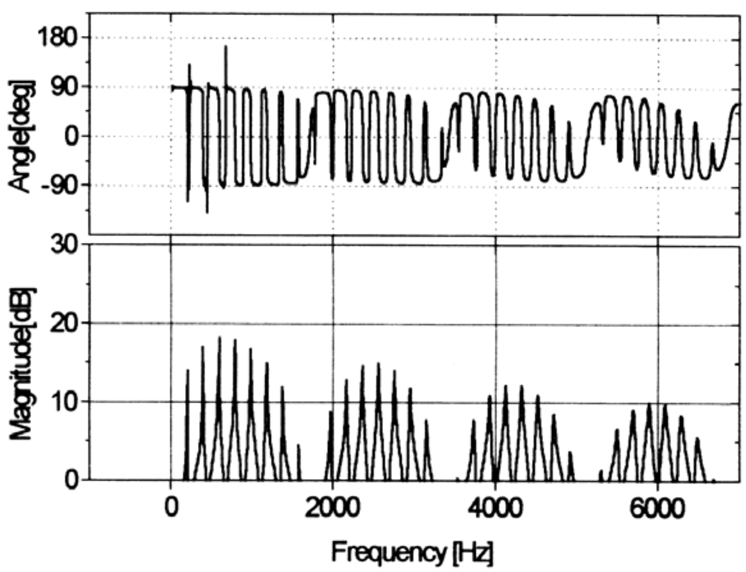


Figures 5 a - c:
The three figures show different approaches for defining the transverse point admittance of the modeled violin G-string (196 Hz; 0 dB = 1 s/kg).

(a) gives the spectrum as obtained with random bow velocities over 8192 time steps, during which the friction remained static. A Hann window is utilized before the FFTs. The noise is quite visible.



(b) shows the same system with a Hann-tailed chirp as the input signal (see text). Also here, the normal Hann window is utilized before the FFTs. The noise is mainly found below 2 kHz. As can be seen, the peak values of the lower harmonics are reduced, compared to (a).



(c): As for (b), a Hann-tailed chirp is used for input signal. Before FFTs of the frictional force and the transverse string velocity, a flattop-Hann window is multiplied with these outputs. The (unity) flat top covers half of the 8192 elements. Some noise is still remaining, particularly in the phase information, but the improvement is obvious.

The main reason for taking such great care when designing the input signal, are the other windows which inevitably follow in the next steps when determining the spectral characteristics of high-Q systems such as a violin string. Any window multiplication in the time domain will be equal to convolution in the frequency domain, i. e., the individual frequency magnitudes will bleed and the phase information be distorted. Minimizing the differences of magnitudes between adjacent frequencies is therefore imperative.

For determining the point impedances of the string, it is the author's experience that a flattop Hann window prior to the FFT, works well in most cases if one places most of the chirp under the flat mid-section of the window (see Figures 5 a-c).

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