The Helmholtz motion

In order to understand how the string is affected by bow gestures, and how the tone color and dynamics can be manipulated by these, it is of vital importance to have a clear concept of how the string "normally" moves under the bow. Given the fact that string waves move with very high propagation speeds on our instruments—even supersonic for the violin E-string: 430 meters per second(!) as compared to a "mere" 340 m/s for sound in air—it should be obvious that there is no straightforward matter to observe subtle details with the naked eye. However, the German physician and scientist Hermann von Helmholtz (1821 - 1894), found a very clever way to retrieve this information, and the findings were described in the Appendix of the second edition of his book "*Lehre von den Tonempfindungen*" published in 1885. (English edition: "*On the sensations of tone*" Dover, 1954, Ref. 1)

By use of a vibrating microscope, Helmholtz was able to employ the so-called Lissajous effect available when a medium has a regular, repeating pattern of movement. This was what he found: At any point along the string, the string is alternating between two different velocities (dependent on stick or slip). The string is not moving as a rounded curve from side to side across its equilibrium line, like a naked-eye inspection might make us believe. Rather, the string shape is composed of two straight lines joined in a relatively sharp kink. This kink (presently referred to as *the Helmholtz corner*) travels *along* the string, back and forth once per period, i.e., twice the string length 440 times per second for an open violin A-string. It is the corner's *trajectory* that draws the rounded outline we are familiar with from watching the string when being played (see the dashed lines in Fig. 1, below).

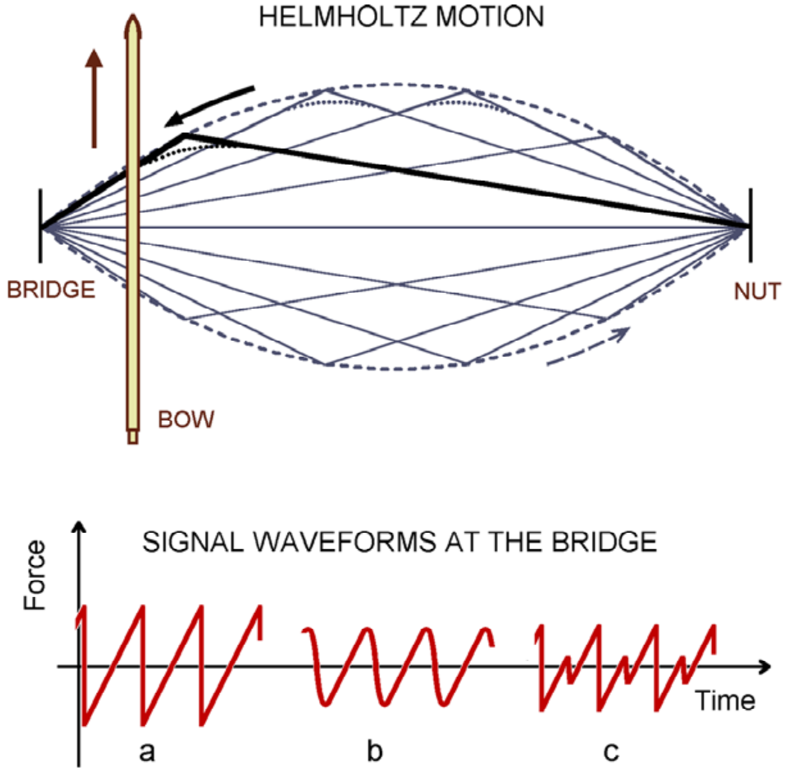

Figure 1: Schematic representation of the Helmholtz motion during an upbow. The Helmholtz corner rotates anti-clockwise along a parabolic path, while the rest of the string is shaped as two straight lines. With a downbow (or an upbow positioned on the nut side) the rotation would have been *clockwise*.

The lower panel indicates (a) the resulting force on the bridge; (b) ditto if the corner gets rounded due to stiffness and losses (see dotted lines below the upper corners of the upper panel); (c) a situation where there are more than one corner traveling (see text).

In the interval when the corner is traveling on the nut side of the bow, the bow has a grip on the string so that the string's contact point follows the bow with equal velocity (the "stick phase"). During the remaining interval (when the corner travels on the bridge side of the bow), the string is rapidly sliding back on the bow-hair ribbon (the "slip phase"). As the corner passes under the bow a quick transition takes place: from stick to slip when the corner is traveling towards the bridge, and from slip to stick when traveling towards the nut.

This implies that during the sticking phase the relative velocity between the bow and the string at their point of contact is zero, while during the slipping phase their relative velocity is equal to the bow speed divided by $-\beta$, where β is the bow's distance from the bridge divided by the length of the active string. (In the literature, β is the commonly used variable name for this ratio.) E.g., if the bow's relative distance from the bridge (β) is 1/8, and the bow speed is 10 cm/s, the string's velocity at the contact point will be -80 cm/s, relative to the bow. And of course: as you move the bow closer to the bridge, the ratio between the bow speed and the relative slipping speed will increase drastically.

So much for the schematic Helmholtz motion…

Where does real life deviate from the representation above? First, take a look at Fig. 2, below:

Friction:

During the steady-state oscillation, friction causes the string to be pulled up at the contact point with the bow-hair ribbon. This distorts the clean, symmetrical string-movement pattern shown in Fig. 1.

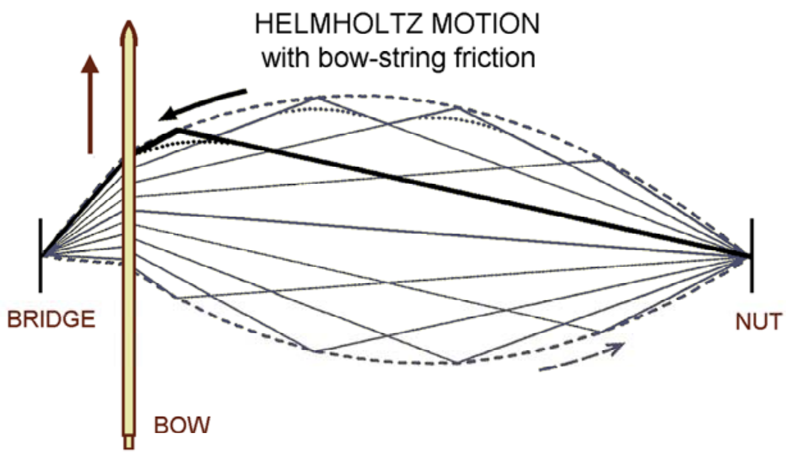

Figure 2: Friction distorts the symmetric parabolic trajectory. In practice, friction causes a pretty constant pull in the bowing direction when the string is oscillating in steady state.

The friction is at all times exerting a force on the string in the bowing direction, whether it be "sliding" (dynamic) or "static". If we look at the classical dry-friction model, the characteristic curve looks more or less as shown in Fig. 3:

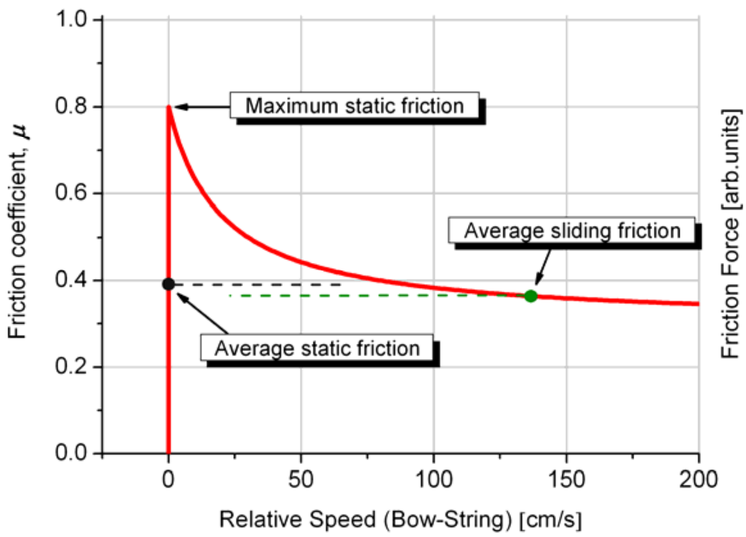

Figure 3: The "classical" dry-friction coefficient curve is vertical from zero up to a maximum value while the two objects are locked in a static grip (sticking). When this maximum value is reached, (say: slightly "surpassed"), friction can no longer keep the objects together, and sliding takes place (the relative speed becomes higher than zero. The faster their relative speed becomes, the lower the frictional resistance between the two will be.

One should notice that static (stick) friction can take any value between zero and the limiting maximum static friction force (which is equal to the "pressure" force times the friction coefficient, μ) while during slip, the friction is the result of the relative speed.

During Helmholtz motion, the *average* static friction value (indicated as a black dot in the coefficient/friction-force plot) will be a trifle higher than the average sliding friction (indicated as a green dot). The reason for this is that at all times a tiny bit of the wave energy leaves the string (e.g. to move the bridge so that sound can be created), and this "loss" has to be compensated for during the sticking phase. The greater the loss, the greater the difference between the two averages. At the very end of the stick phase, there will also be a quick excursion to the maximum friction force level before sliding starts. This spike of force is sharpening any rounding of the rotating corner that might have occurred during the last round.

All this being said, the classical dry-friction model has proven not to be very exact when it comes to rosin, bow hair, and strings. My article "How does rosin affect sound" might give you some insight in how things probably are (otherwise, read the ground-breaking, but more technical article by Smith and Woodhouse, Ref. 2). This, however, does not imply that the simpler explanation above fails to provide some insight into the basic features of friction dynamics. You should also be aware of the difference between the relatively smooth friction force during *steady-state* Helmholtz motion, and the *rapidly changing* friction force during attacks and other transients. E.g., when you start a tone with the bow on the string, there is initially a slow buildup until the maximum friction force is reached, and after the first release has taken place, quite a few major friction-force peaks will occur before the Helmholtz pattern is established.

Secondary waves

In addition to the frictional "pull-up" described above, any wave generated during the stick interval—be it invoked by the bow or the bridge—will be locked in and reflected between these two barriers (see red lines in Fig. 4, below). Their fate is not completely known, but they don't seem to make a great impact on the sound, because they will be operating at oscillation frequencies where the spectrum of the Helmholtz-moving string has some natural, quite significant notches. However, these waves should be observable if you put a high-speed camera next to the actual part of the string. When I refer to these waves as "locked in" I am exaggerating quite a bit, because *during the slip interval* they will not only escape, but even get *amplified* as they pass the bow in the direction of the nut, due to the negative resistance characteristics of the rosin (i.e., less resistance as the relative speed increases). But, as I already suggested, you can pretty much forget about these "secondary" waves—until some clever researcher claim their importance.

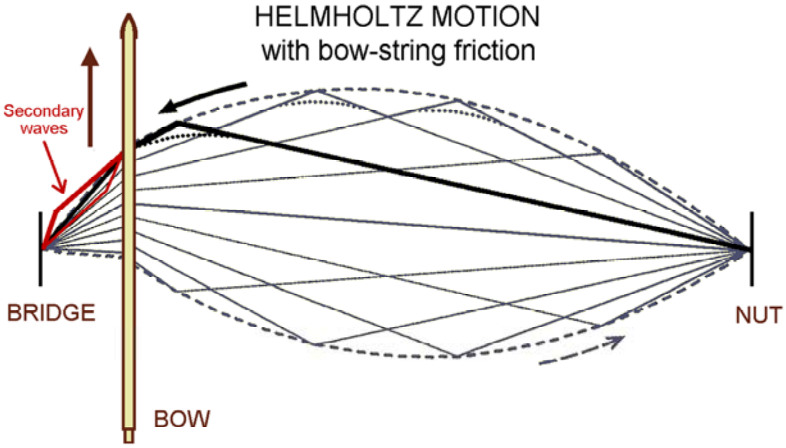

Figure 4: Secondary waves are locked in between the bow and the bridge during the stick interval (see red lines near the bridge).

Torsional waves

There exist, however, another kind of waves that do have some impact on the sound, and even more so, on the playability. These are called torsional waves because they are created by the bow twisting the string, touching the string tangentially at its surface (see Fig. 5, below).

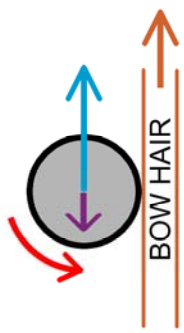

Figure 5: The creation of torsional waves. When the bow is in static (non-sliding) contact with the string surface, it pulls the string surface up with the same velocity (brown and blue arrows). However, if the string is free to rotate (red arrow), the string *center* will not follow entirely, but experience a reduction in speed (violet arrow) due to the string's rolling back on the bow-hair ribbon. The total motion of the string center will thus be depending on the ratio of the respective torsional and transverse wave resistances.

Instead of following the bow entirely during the stick phase, the string will roll somewhat back on the bow-hair ribbon. If no other waves are present, the speed with which the string is rolling back depends on the bow speed, of course, but also on the ratio of the torsional and transverse characteristic wave resistances: Had they been equal, the string center had been taking only half the bow speed (ignoring any reflection returning from the bridge or nut); the rest had been lost in the string's rolling.

However, in the same way that moving the string center causes transverse waves to be excited in both directions, rolling the string will cause *torsional* waves to propagate in both directions away from the contact bow's point. As far as we now, they will not produce (noticeable) sound when reaching the bridge, but they will be reflected there and return to the bow with opposite phase. There, a new rolling will take place, and the string's center will move even if the sticking bow has been stopped in the meantime.

The propagation speed of torsional waves is, in practice, always higher than the propagation speed for transverse waves, ranging from about twice the speed (e.g. for homogeneous *gut* strings) to slightly less than eight times the speed (e.g., for homogeneous *steel* strings). The propagation speed of torsional waves is for the good part independent of the strings' tuning, but for strings with wire-rope core, the degree of core twisting influences the propagation speed in such a way that high-level twist gives higher propagation speed, and vice versa.

Torsion implies that waves can pass the bow even during the stick phase. Because of that, there will always be a certain amount of ripples on the "straight-line sections" of the Helmholtz figure.

Durations of the stick-slip phases

In the schematic Helmholtz motion, the slipping phase is equal to β times the total period, T, where T is the time for a full corner roundtrip, and the inverse of the tuning frequency. However, when the corner gets significantly rounded, as it does when the bow force is low compared to the bow speed, like in "flautando" playing, the slipping phase can extend far above βT. This is actually very good when imitating the flute, because the hiss that is mainly created during the slipping phase (due to frictional irregularities during the string's slide on the bow hair) is now expanded in time and the "breathy" sound thus stand out as more prevalent.

Summing up:

The schematic Helmholtz figure provides the most important features of the bowed-string dynamics during "normal" tones, and has been invaluable in understanding what really happens under the bow. However, while the schematic Helmholtz pattern provides the skeleton, there are a few more features on top of it, shaping the full body, so to speak: (1) the more or less constant friction; (2) the secondary waves; (3) the torsional waves. You should also add to the picture string stiffness, and internal and external losses. Internal (molecular) losses due to string bending provide a transition from mechanical energy to heath energy—warming up the string. Such losses have to be compensated for by the bow supplying new energy.

References:

1: H. v. Helmholtz, *On the Sensations of Tone* (Dover, New York 1954).pp Original publication: "*Lehre von den Tonempfindungen*" Braunschweig: Vieweg (1862).

2: J. H. Smith and J. Woodhouse, "The tribology of rosin" Journal of the Mechanics and Physics of Solids **48**, 1633-1681 (2000).