String stiffness

The resistance the player feels when starting the string, is composed of a number of factors and phenomena, the most important one being the "characteristic transverse wave resistance", which can be calculated as the as the product of the string's transverse wave-propagation speed, and mass per unit length (giving the dimension "mass per second"). In addition, we have:

Bending stiffness

Every string has a certain bending stiffness, i.e., resistance against being bent. This, of course, has some implication in terms of how the Helmholtz corner will look, how sharp it is going to be under the bow. It also has implications concerning how fast waves of different frequencies will propagate on the string; higher frequencies will travel faster than lower ones, providing a neat way to measure the approximate stiffness of different strings. When plucked, the overtones will not be exact integer multiples of the fundamental frequencies (they will be "inharmonic"), the stiffer the string, the more the overtones will deviate from the harmonic relation present when the string is bowed. Norman Pickering measured just that, and he reported some interesting numbers in his paper "Physical properties of violin strings" (Ref. 1). Of the most popular violin strings of that time, he reported the stiffest G-strings to be (with decreasing stiffness): Jargar (steel core), and Eudoxa, Kapland, and Super sensitive (all with gut core)—while Kaplan (synthetic core), Dominant (Perlon core), and Gold Label (gut core) came out as the most flexible ones. For the D, A, and E strings, the results were (presented in the same order) D-strings: Kaplan (synthetic core) and Jargar (wound steel core) on top, to Kaplan (silver wound core and gut core) and Dominant (Perlon core) at the bottom; A-strings: Jargar (wound steel core) and Dominant (Perlon core) on top, while the rest seemed pretty flexible; for E-strings the difference was insignificant.

Maybe it is a bit surprising that the gut-core strings came out so stiff? Unfortunately, Pickering does not deliberate on that fact. Anyhow, you should read his full report!

But, what does the stiffness imply in terms of playing? If we start with the plucked string, I would have thought that I should be able to hear the difference between a stiff string and a flexible string, the first one sounding more metallic due to greater inharmonicity. I don't. That is, I would most probably be able to hear the difference between two different brands, but when I simulate two strings where their stiffness is the only difference, I can't. However, when I add sustain to the simulation, I can certainly distinguish between them. But, with the rapid decay of a plucked violin string, I can't say that the one "plop" is more metallic than the other… On a cello or double bass, with greater sustain, it is different.

So to arco: It is only in pizzicato the inharmonic overtones are at liberty to produce a "metallic sound", simply because the waves are free to run at different speeds with no synchronizing element. When bowed, however, their phases gets locked every time a slip or grip takes place: the waves arriving early at the bow will simply have to wait, so that they all start with equal conditions every time there is a transition from stick to slip, or vice versa. This implies that there is a very small amount of additional work to be done when a stiff string is being played. One may be able to feel a trifle higher "resistance" with the stiffer string.

String inharmonicity, which is dependent on the fourth power of the string's core diameter, is best reduced by keeping the diameter as low as practically possible.

Resistance to elongation

However, even more noticeable than bending stiffness, may the resistance to string elongation be. If you want to play a series of rapid notes, you would prefer a string that is not too compliant in the bowing direction. If it moves quietly with the bow like a rubber band before each tone onset, your action is slowed very much down. Some lower strings of each instrument for the violin family may possess this undesirable property. In harp and guitar, where the strings are plucked, this effect is rather crucial: you don't want individual strings of different "feel", as the attacks would be much harder to control with respect to both timing and sound. Of course, the string's tension plays a role here—for solo playing on the double bass I used to tune up normal E and A orchestra strings to solo tuning: F# and B, to get the extra tension, and thus more rapid action—but more important is the so-called "Elasticity modulus" (often referred to as "Young's modulus") of the core material. The elasticity modulus determines how much a material will stretch when pulled with a certain tension. This varies quite much between different core materials. The key words here are:

Strain = $\Delta L/L$ (i.e., elongation per unit length);
Stress = Tension/Cross section of core (also referred to as "tensile stress");
Elasticity modulus = Stress/Strain =
Tension × String length / (String's prolongation × Cross section of core).

So: the greater the Elasticity modulus, the greater the resistance to stretching. Pickering (Ref. 2) presents a table of elasticity values from some well-known brands of violin strings. Not surprisingly, the steel core is by far the material with the greatest Elasticity modulus, some 3 to 5 times higher than gut at the bottom end of the scale, with the synthetic (e.g., Perlon) a little above.

Torsional "stiffness"

Because the bow is exciting the string tangentially, not through its center, part of the energy will cause a rotation/twist of the string in the direction of bowing. For the most part this energy is lost (dissipated in the string), therefore the string manufacturers mostly try to limit this action by twisting the core, covering the core inside a braided hose, or having more layers of windings with turns of opposite directions (see figures of Ref. 3, in Library). This last method is commonly used in wire ropes for sailboats, where it is crucial that the ropes are not twisting. With successive layers of turns of different orientation, half will be tightened while the other half is loosened during an attempt to twist the rope.

The stiffer a string is in this respect, the higher the torsional propagation speed and the torsional characteristic wave impedance will be, because these two follow each other proportionally as long as other parameters are kept constant. Similar to transverse string elongation, there might be a dead-zone torsional rolling before any tone is created.

I remember substituting in an American college orchestra during my U.S.A. studies in -68: I was given a bass with rather ancient strings. The E-string was furnished with a metal-wound gut string—mostly gut, though. The string was almost as thick as my little finger, and it sometimes made a full rotation before any sound was heard. The sound was often accompanied by some bow hairs being torn off, having got caught between rather open windings of the copper-alloy wire. I presume quite a great part of the classical standard repertoire had never been realizable with that string design.

Increased “stiffness” closer to the bridge

Although a string feels progressively “stiffer” as you move the bow closer to the bridge, this is not really a property of the string, but a matter of an increasing number of (negative) reflections at the bridge, trying to send the string in the direction opposite to the bowing. The number of reflections is inversely proportional the bow-to-bridge distance. It should be noticed that torsional waves, not only transverse waves, reflect in this manner. Here, an important parameter is the torsional wave propagation speed, as it determines how often such reflections will meet the bow. For example, for homogeneous gut strings, there will be about two torsional reflections for each transverse one; for synthetic core, typically 5 to 6, and for homogeneous steel strings, between 7 and 8. The more torsional reflections, the stiffer the string will feel.

In Fig. 1 below, the resulting friction force during the quiet part of an attack (i.e., before the first slip) was simulated with the bow starting from the string, with constant acceleration. The violin string’s ratio of torsional and transverse propagation speeds was programmed to be 5.5, while the ratio of their respective characteristic wave resistances was 2.5.

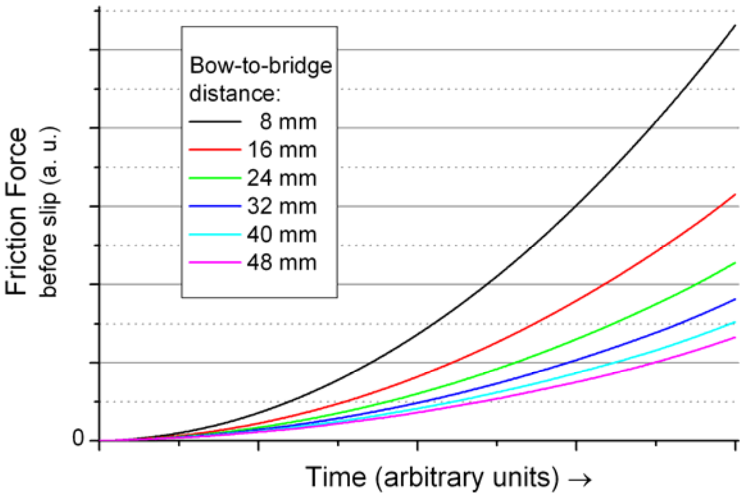

Figure 1: As the bowing position is moved towards the bridge in steps, the friction force (“resistance” or “stiffness”) felt by the player during an attack increases dramatically. Once the Helmholtz motion has started, the friction forces get lower and the string becomes easier to control close to the bridge. The curves show friction-force buildup before the first slip, all with the same bowing.

Let us repeat the last experiment in order to evaluate the influence of torsional propagation speed on the friction force. This time with a fixed bowing position, but with three different strings: homogeneous gut, synthetic core, and homogeneous steel. *The only parameter* *changing is the torsional propagation speed* (see Fig. 2). Forces raise most quickly for the steel string, closely followed by the synthetic core, and with gut some 12% below the steel. Had elongation parameters (strain) been included in these simulations, the steadily increasing steepness of the force curve had been counteracted by the string's compliance: the "gut" string had been giving in first (causing a noticeable reduction of the force buildup), quickly followed by the "synthetic-core", and finally, but to a lesser degree, the "steel" string. The three curves would have been somewhat more separated.

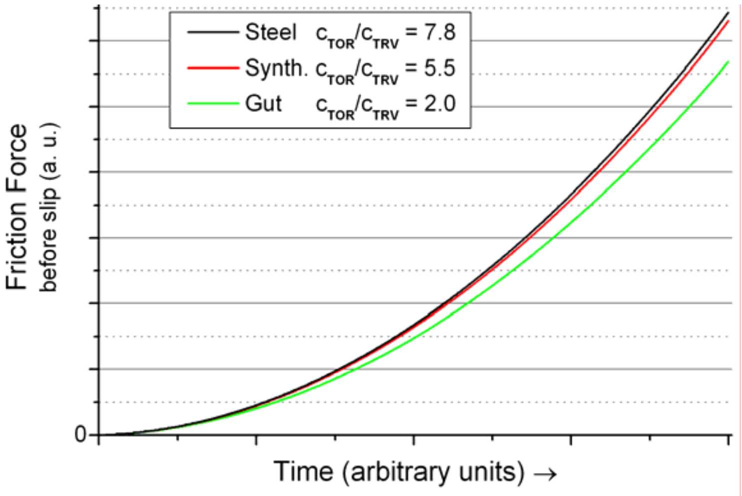

Figure 2: Example of friction-force buildup with three strings of different torsional propagation speeds (other parameters kept identical). The homogeneous-steel and the synthetic-core strings are almost similar as far as buildup is concerned, while the homogenous gut string lies some 12 % below the steel string.

After expiration of the onset transient, when the Helmholtz pattern is established, the string is easier to control near the bridge. For this reason many players start the bow stroke with the bow at a distance, and later bring it to the appropriate place for the note in question. In the same manner it is quite common to pull the bow away from the bridge for bow changes.

What does all this imply for the sound?

Here, I have to rely on Pickering again (Ref. 4). So far I have only been touching on some playing properties. We should expect that the strings with the highest bending stiffness (steel strings) would provide the greatest variation in tone-color possibilities, because these would give the greatest rounding of the Helmholtz corner at low bowing pressure. At the other end, because steel strings can master also the highest bow pressures (without getting choked), the corner can be sufficiently sharpened to give high brilliance. Pickering's article confirms this, and concludes further with the following statement: "Gut cores show the least [spectral] variation, and synthetic polymer cores are somewhere between".

Ref. 1: N. C. Pickering, "Physical properties of violin strings" Catgut Acoust. Soc. J. **44**, (1985). (See "Library")

Ref. 2: N. Pickering, "Elasticity of violin strings" Catgut Acoust. Soc. J. **46**, 2 - 3 (1986). (See "Library")

Ref. 3: I. Firth, "Construction and performance of quality commercial violin strings" Catgut Acoust. Soc. J. **47**, 17-20 (1987). (See "Library")

Ref. 4: N. Pickering, "String tone quality related to core material" Catgut Acoust. Soc. J. **1, No. 5 (Series II)**, 23 - 28 (1990). (See "Library") Notice that the dB levels referred to in this text are measured as string *movements* with no direct reference to how the sound would come out on a violin. High-gauge strings (i.e., strings with high characteristic wave impedance) would most likely sound louder than low-gauge strings when mounted on a violin, even if they achieved the same dB values in these tables. The force acting on the bridge is a function of transverse string velocity × characteristic wave resistance. In these tables only the first term was accounted for.