Double stops

When playing double stops, notice that the bow pressure should be unequally distributed between the two strings involved. Generally, the string producing the lowest pitch should be given the highest pressure. Here is why:

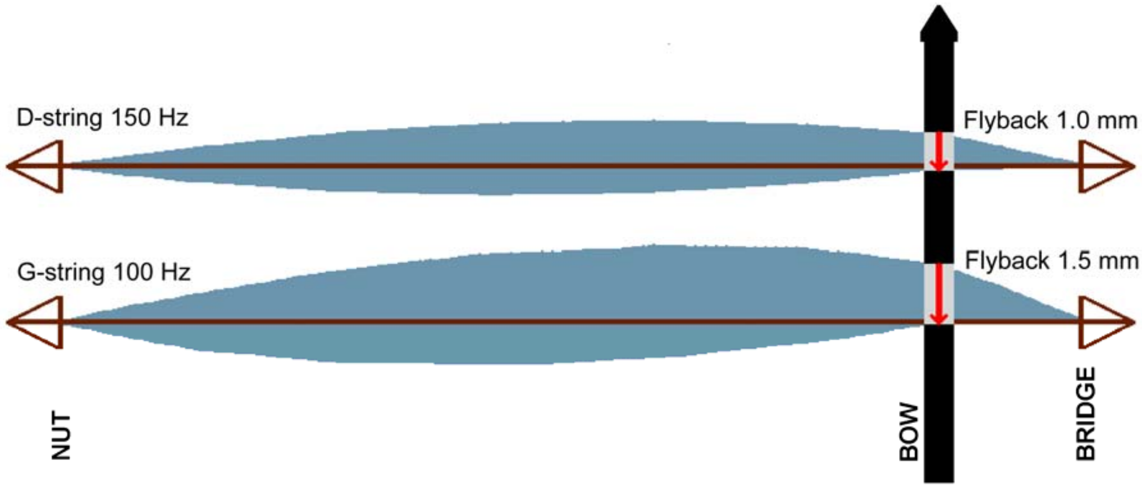

Figure 1: Perfect fifth. The D- and G-strings slip on the string 150 and 100 times, respectively. The string with the lowest pitch shows larger amplitude. The bow's position relative to the active string length (β) is about 1/7 for both strings, counted from the bridge.

Imagine the two open cello strings G_2 and D_3, swinging with approximately 100 and 150 Hz, respectively. Ideally, the two strings should have been bowed with two different speeds in order to produce comparable tone colors, which of course in practice is not possible. However, real life implies that the upper string has to slip back on the bow-hair ribbon 150 times per second, while the lower string makes 100 only. If the bow is moving with a speed of 15 cm/second, each flyback on the G-string (100 Hz) will thus be one-and-a-half millimeter, while on the D-string (150 Hz) the flyback will be one millimeter. (Frequency × Flyback distance = Bow speed.) Even if these two strings had the same wave resistance[1], the G-string will require a higher friction force than the D-string to be pulled out to sufficient amplitude, particularly during the onset transient.

In the case of playing octave, where the D-string is stopped at the forth (G_3) the raise in frequency will demand a further reduction of bow pressure, since the flyback now must be reduced to a mere 0.75 mm, i.e., the half of the flyback on the on the open G-string (see Fig. 2):

[1] "Wave resistance" or "*characteristic* wave resistance" is the operative word here. When the string is given a certain tension, waves will travel along it with a speed (in meters per second) that is equal to the square root of the tension (in Newton) divided by mass (in kilograms) per meter. That is, Speed = √(Tension/Mass per meter). The characteristic wave resistance is then simply Speed × Mass per meter, which has the dimension kilograms/per second. **The necessary friction force required to create a certain wave on a string is proportional to its characteristic wave resistance. Therefore, tension, string thickness or string weight *alone* gives no indication of how great the force must be in order to move the string to a given wave amplitude. Only wave resistance does!**

In many cases, you might want to emphasize the top string, because it is carrying the melodic line. However, you may easily get into trouble if you give it more weight: The string may get choked, with the consequence that the pitch drops. In such cases it is not a smart idea to compensate the pitch drop by adjusting the left-hand finger. The sound will indisputably be best if the two strings are bowed with *different* bow pressures to produce equal timbres (brilliance). A rich collection of audible difference tones will appear, which gives the double stop a depth and warmth that otherwise would be lost. Double stops always sound best when the two tones are of equal timbre quality; and *bow pressure* is the tool for controlling it. However, strings are less prone to pitch flattening when bowed closer to the bridge; a fact that can sometimes come to rescue when a good balance is difficult to achieve.

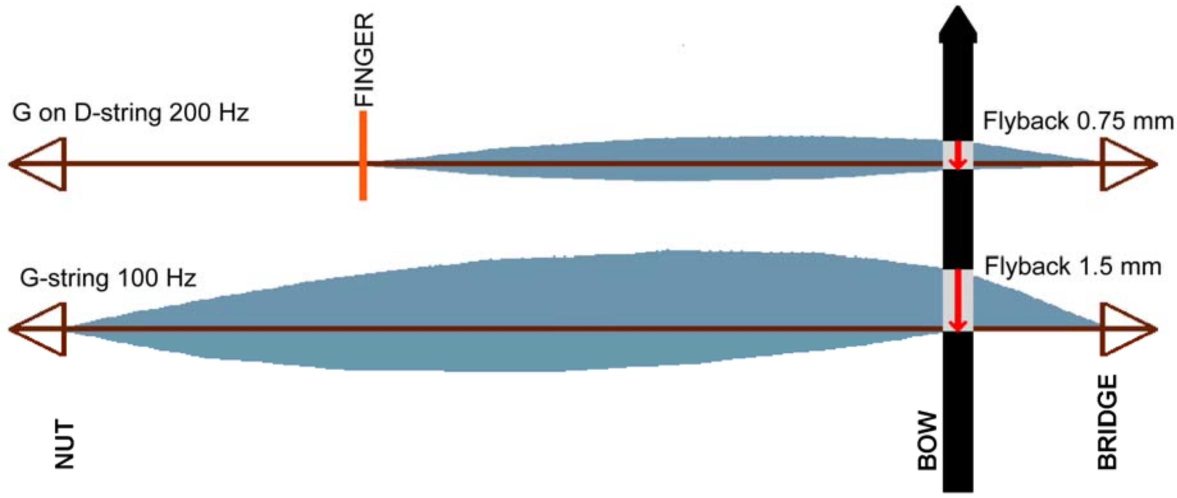

Figure 2: Octave. Example of the upper (D-) string stopped at G_3. Now the frequency ratio is increased to 2, demanding further bow-pressure reduction at the upper string, to reduce string flyback.

With typical string parameters, the G- and D-strings do not possess equal wave resistances. On a cello, the G-string resistance is typically 40 to 45% higher than the resistance of the D-string. Comparable values for violin and viola G- and D-strings are 50 to 100%, and around 50%, respectively. With a possible exception for the violin E- and A-strings, the wave resistance increases as the strings go heavier. Higher wave resistance requires higher friction force (or bow pressure, if you like). To sum all this up in a simple rule of thumb, we could write:

***The approximate bow-pressure ratio between the two strings in a double stop should be the inverse ratio of their sounding frequencies*.**

Or simpler:

***Always more weight on the lowest-sounding string*!**

Some typical intervals and their corresponding frequency ratios (highest frequency divided by lowest frequency) are: Perfect fifth: 1.5; Major sixth: 1.7; Octave: 2.0; Major tenth: 2.5; Octave + Major fifth 3.0; Two octaves: 4.0.

E.g., if the frequency ratio is 2, i.e., an octave, the string that produces the *lower tone* should have about twice the weight of the other one. (*Notice that this is true even if the highest pitch is produced on the lowest-tuned string*!) In the case of major tenths, the bow-pressure ratio should be approximately 2.5, and so on. It goes without saying that this becomes more critical the larger the interval is. For lower-pitched instruments, like the cello and the double bass, intervals greater than one octave might also be broken—starting with the lower string alone—to avoid harsh, scratchy, and lasting onset transients. As always is the case: one has to find the right execution by ear—but it doesn't hurt to know where to search!

Intonation

Intonation is an important issue when playing double stops. While string players usually have some freedom with respect to exact intonation, due to vibrato and unison group interference, the situation is turned somewhat around for a string soloist playing double stops. Like classical wind players always try to use *just intonation* whenever their groups are exposed (and it is applicable to the chord), string players should aim at just intonation for their double stops when applicable. The table below compares just intonation to equal-tempered intonation[2] (intervals not listed below are not applicable for just intonation). Important: these intervals do only apply to chord playing and may be different in melodic single-tone playing.

Interval (upwards)	Trend	Cent
Minor Third	Augment	+16
Major Third	Diminish	–14
Perfect Forth	Diminish	–2
Perfect Fifth	Augment	+2
Minor Sixth	Augment	+14
Major Sixth	Diminish	–16

The reason why just intonation is recommendable is that they sound less dissonant, and they have a stable, repeating wave pattern that invokes clearly audible difference tones, which bring depth to the sound[3].

Typical places where awareness of all the above might prove beneficial, are the final chords of Bach's Solo Suites, Sonatas, and Partitas.

I am sorry to say so, but in my experience, the topic of double stops is the least understood, and hence most often erroneously executed among professional string players.

[2] Additional octaves make no difference to the table values.

[3] A good example of this can be found at the page "About myself", in my Greensleeves video . Just starting at the second half, I play a variation with harmonics, several of which occurring in double stops. The lowest-sounding interval is a major tenth (with pitches D_2 and $F\#_3$, played on the lower two strings). Because the upper tone is a natural harmonic, and its pitch thus fixed, I play the lower tone deliberately a little sharp (ideally, +14 cent) to shrink the interval (see the table). I also give considerably more weight to the lower string. Without these adjustments, the sound would have been dull and much less satisfying.