THE BOUNCING BOW:
An Experimental Study

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The bouncing bow as used in rapid spiccato *and* ricochet *bowing has been studied. Dynamical tests were made by monitoring the motion of the bow stick and the bow force history when the bow was played by a mechanical device against a force transducer as a substitute to the string. The action of the bow stick was also investigated by modal analysis. Bows made of wood, fiber glass, and carbon fiber composites were studied and compared, as well as a bow which was modified from the normal (concave) shape into a straight stick.*

The advanced bowing styles called rapid *spiccato* (*sautillé*) and *ricochet*, in which the bow bounces off the string between notes by itself, can be performed at high rates, between 8 and 13 notes/s (sixteenth notes at M.M. ≈ 120-190 quarter notes/min.). The dynamics of the bow plays an important role in these bowing styles, and differences in the action between bows are easily recognized by professional players. All instruments in the string orchestra can perform these rapid bowings despite the large differences in scaling between the instruments at the extremes, the violin and the double bass.

Bounce Mode

A low-frequent "bounce mode" of the bow is of primary importance for the rapid spiccato (Askenfelt 1992a). With a light bow hold which is necessary for a rapid spiccato, the bow can be considered as pivoting around an axis roughly through the cut-out in the frog (thumb and middle/index finger at opposite sides). The moment of inertia J_x of the bow with respect to this axis and the restoring moment from the deflected bow hair (and string) defines a bounce mode with a frequency which is dependent on the tension T of the bow hair and the distance r_s between the contact point with the string and the pivoting point (see Figure 1). Assuming that the stick behaves as a rigid body and that the bow hair of length L stays in permanent contact with the string, the bounce mode frequency f_{BNC} against a rigid support is given by

$$f_{BNC} = \frac{1}{2\pi}\sqrt{\frac{T\left(\frac{r_s}{1 - r_s / L}\right)}{J_x}}$$

The shape of the bounce mode is illustrated in Figure 6 (mode #1). Typical values of f_{BNC} versus the distance r_s from the frog for a violin bow are shown in Figure 2. The calculated values range from 6 Hz close to the frog to about 80 Hz at the very tip. Measured data for a real bow, given for three different hair tensions, follow the calculated values closely up to about 2/3 of the bow length. For the remaining third the increase in f_{BNC} is much less than the calculated case for a stiff stick, reaching only 40 Hz at the tip. The difference indicates that a flexing of the thin outer part of the stick and the motion of the head (see Figure 6) influences the transversal bow stiffness in this range (cf. Pitteroff 1995). The rapid spiccato is played well inside this part, typically in a range between the midpoint and 2 -3 cm outside the middle (towards the tip). In this range f_{BNC} will be between 13 - 15 Hz, and the stiff bow stick model predicts the bounce mode frequency accurately. In real playing, the compliance of the string lowers the bounce frequency by 1 to 2 Hz, depending on the bow-bridge distance (1 - 4 cm). The lowering in frequency is essentially the same for all four strings on the violin.

Another effect encountered in real violin playing is the tilting of the bow. This will also lower the bounce rate. When loaded sideways, more and more individual bow hairs will successively be

Figure 1 ■ Geometry of the bouncing bow

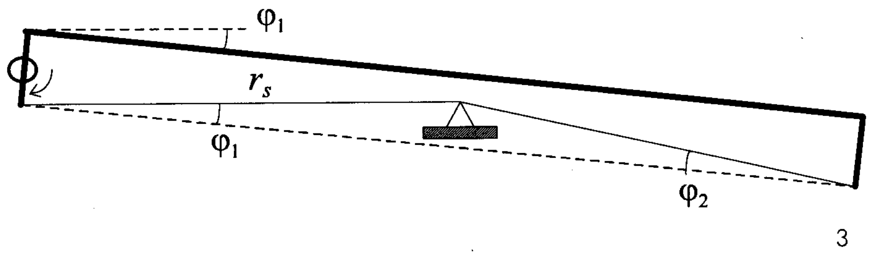
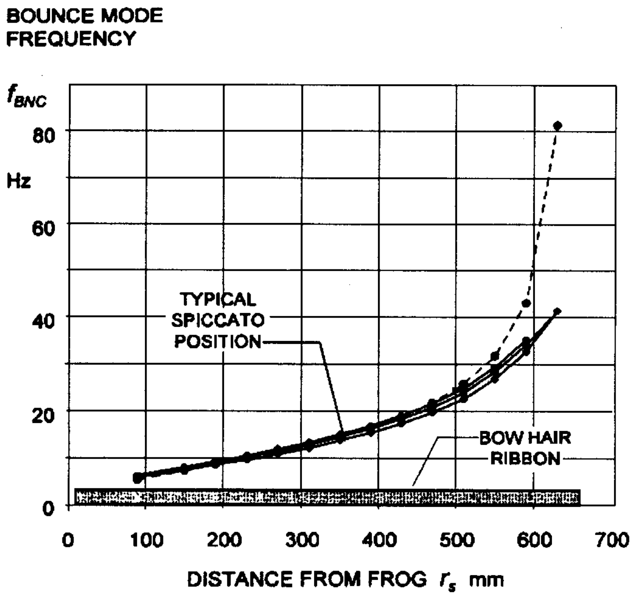


Figure 2 ■ Bounce mode frequency f_{BNC} against a rigid support for a violin bow pivoting around an axis at the frog. Calculated values (dashed line) and measured (full lines) for three values of hair tension ("normal" = 55 N and ± 5 N corresponding to ± 1 turn of the frog screw). The hair was kept in permanent contact with the support.



brought into play as the bow contacts the string, and the "effective compliance" during a bounce with a tilted bow will be higher than for an impact with the hair flat on the string. At a "best spiccato position" about 2 cm outside the midpoint, a typical tilting of 30° will lower the bounce rate by 1 to 2 Hz compared to the case with the hair flat on the string, the exact value depending on the particular bow and tension. In order to recover the bounce frequency with the tilted bow, the playing position must be shifted 3 - 5 cm towards the tip. Apart from giving softer attacks because of a slower build-up in bow force, tilting gives the player access to a fast control of the bounce rate. The very rapid spiccato must always be played with the hair flat on the string in order to keep the bounce rate as high as possible.

The player's preferences for the bounce frequency, as reflected in the setting of bow hair tension and choice of contact point with the string, seems to be precise. A professional string player was asked to set the optimal conditions for a rapid spiccato at 11 Hz (sixteenth notes at M.M. = 160) for six bows of very different quality. The selection included an excellent French bow (*F. Voirin*), a fiber-glass bow, and a student's bow of very poor quality. The task consisted of tightening the bow hair to a suitable tension and choosing the best contact point. The bounce frequencies, as measured at the chosen contact points (from 2 cm outside the midpoint to 3 cm on the inside) stayed between 13.7 and 14.5 Hz. For an individual bow, this frequency interval corresponds to an adjustment of the tension by typically ± 3 N (or about ± ½ turn of the frog screw) around a normal playing setting in the vicinity of 55 N. The same variation range in hair tension seems to be a reasonable estimation of the spread in preferences among players for the optimal tension of a particular bow.

The Q-value of the bounce mode is of the order of 30 - 50 when measured with the bow pivoting freely around a steel axis in a brass bearing at the frog (see Figure 1). When simulating playing conditions by letting a string player hold the bow in a normal bow grip while still supported on the support, the Q-values lowered to about 15 - 20.

Take-off and Flight

The bow leaves the string when the contact force has decreased to zero. At that moment, the angular acceleration is high enough to make the "inertia moment" $J_x\, d^2\varphi_1/dt^2$ match the "bowing moment" M_B supplied by gravity and the player's bow hold. The motion during the flight time is depending on the character of the bowing moment M_B. For a horizontal bow, gravity will give a constant moment independent of the bounce height and a flight time proportional to the angular velocity at take-off. The bow will bounce with an increasing rate when dropped against a support. At a typical spiccato position on a violin bow, this occurs at an initial frequency of about 7 - 8 Hz for a reasonable starting height (about 1 cm), approaching f_{BNC} asymptotically as the bounce height decreases.

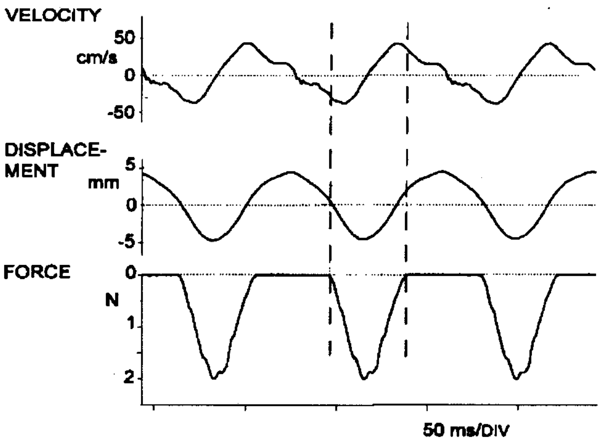
An additional restoring moment during flight, supplied by the player, enables a faster spiccato than the gravity-controlled (much like a bouncing ball which not is allowed to reach its maximum height). Due to the compression of finger tissue, this additional moment is probably of a "springy" character, increasing with deflection angle φ_1. For the cello and double bass, which are played with the strings nearly upright, the contribution from gravity is much reduced. In all, the two different restoring moments acting during string contact (deflected bow hair) and flight (the player's bow hold), respectively, will roughly give a oscillating system which switches between two slightly different spring constants at take-off and landing.

Point of Percussion and Stick Bending

The "point of percussion" (PoP) is a parameter which often is referred to in discussion of bows. When hanging vertically, pivoted at the cut-out in the frog, the bow will oscillate with a low frequency, typically 0.72 - 0.75 Hz for a violin bow. This corresponds to the motion of a simple pendulum (point mass) with length l_{PoP}. For a compound pendulum consisting of a straight rod of uniform cross section, l_{PoP} will be 2/3 of its length. The violin bow is close to this case with a total length from the pivoting axis at the frog to the tip of typically 680 mm and l_{PoP} between 450 and 480 mm. A double bass bow will have PoP relatively closer to the tip.

An external force at PoP (such as when the bow lands on the string) will give no transversal reaction force at all at the player's bow hold. Forces which are applied inside and outside PoP respectively, will give reaction forces with opposite signs. Spiccato is always played well inside PoP (about 10 cm), and the impact force during string contact and the corresponding reaction at the pivoting point will tend to bend the bow so that the stick flexes upwards when referring to a horizontal bow (bending away from the hair). These forces will try to straighten out the bow maker's downward camber (concave shape). The accompanying lengthening of the stick would increase the tension of the bow hair, by bringing the end points of the hair farther apart (Pitteroff 1995). This effect is contradicted, however, by a decrease in the angle of the tip (pointing upwards), and the net change in hair tension during contact can probably not be determined without direct measurements.

Figure 4 ▪ Registrations of bow motion in rapid spiccato when a violin bow is driven by a shaker at 12.0 Hz (sixteenth-notes at M.M. = 180). The driving amplitude corresponds to *mf* - *f* level when playing on a violin. The registrations show the contact force against a rigid support 2 cm outside the midpoint, and velocity and displacement of the stick at the position of the support. The vertical dashed lines indicate the contact duration.



Mechanical Spiccato

The rapid spiccato can be reproduced in the laboratory (see Figure 3). The bow is supported by a fixed steel axis in a brass bearing mounted in the cut-out in the frog and the hair is resting on a force transducer at the normal position for rapid spiccato, just outside the midpoint. A steady bow force of about 0.5 N is supplied by gravity and a spring, simulating the player's "bowing moment." The bow is driven by a shaker with a sinusoidal motion, which for each cycle gives a downward push on top of the stick at the normal position of the index finger via a hook and an interfacing piece of rubber. The duration of the push can be varied by adjusting the vertical position of the steel axis. The motion of the stick is measured by a miniature accelerometer on top of the stick at the playing position.

A typical registration of the motion of the stick and contact force is shown in Figure 4. The bow is driven at a spiccato rate of 12.0 Hz, with the driving adjusted to give a peak contact force of 2 N. Contact force and stick displacement are in phase. After an off-string flight, contact between the hair and support occurs with the stick moving downward with a velocity of about 25 cm/s. At that moment, the stick displacement at the contact point is close to zero (equilibrium position). The motion of the stick in this example is considerable with an amplitude of about 5 mm, corresponding to *mf*-*f* level when playing on a violin. A forceful spiccato may drive the stick almost down to the hair, giving a stick amplitude of about 9 mm.

The bouncing bow in spiccato can be viewed as a mechanical series circuit with one restoring moment during string contact due to the deflected bow hair, and a second during flight supplied by gravity and the player's bow hold, acting on the moment of inertia J_x. It follows that the duration of the flight and the duration of string contact not at all need to be equally long (see Figure 4). The flight time may well be extended in order to increase the duration of the free decay of the note. The contact part must, however, remain long enough to ensure a forced damping of the string at the end of the note, and a sufficient buildup of the string amplitude of the note which follows (Guettler and Askenfelt 1998).

Figure 3 ▪ View of the experimental setup for a mechanical spiccato. A hook fastened to the shaker contacts the upper side of the stick via a piece of rubber, thus simulating the player's index finger. Gravity in combination with the spring gives a static bow force of about 0.5 N.

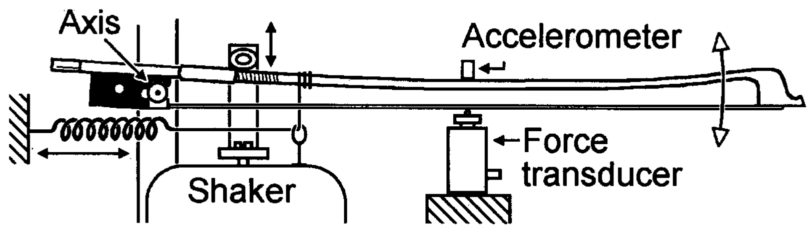
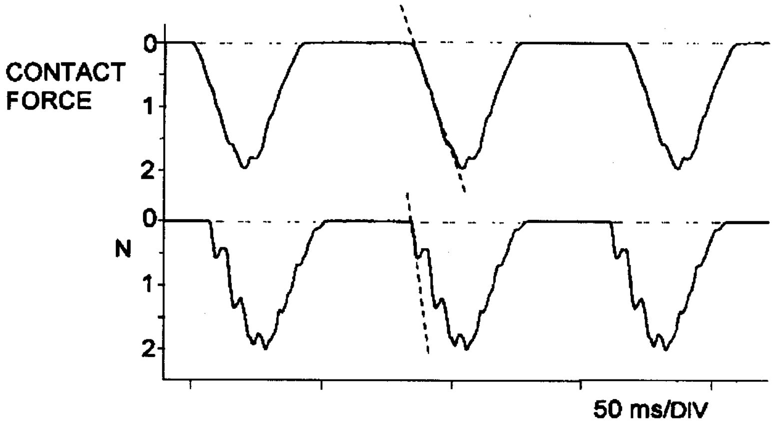


Figure 5 ▪ Comparison of the contact forces against a rigid support for a violin bow driven in spiccato at 12.0 Hz (*top*) and 13.0 Hz (*bottom*). The peak force was kept constant at 2 N. Notice the difference in strength of the 150-Hz ripple. The dashed lines show the initial slope in contact force, corresponding to 0.12 and 0.23 N/ms, respectively.



The driving condition which seems to resemble the action of a string player best is to drive the bow just below the bounce mode frequency (about 2 - 3 Hz lower). The "finger" of the shaker (the player's index finger) gives a down-ward push on the stick close to its upper turning point. Depending on playing style, the duration of the push may extend into the contact part with the string, and thus be longer than a quarter of the spiccato period, but it may as well end before the bow contacts the string.

Contact Force

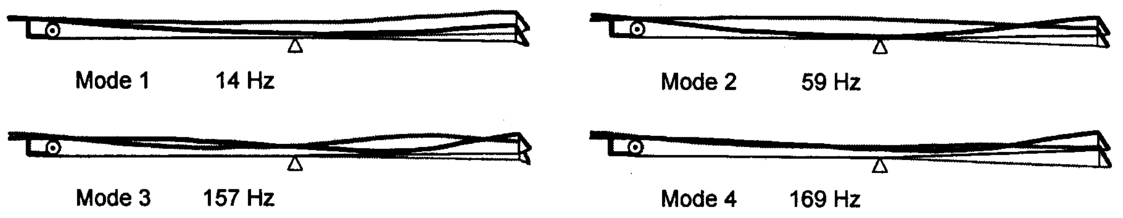
The contact with the string lasts approximately half the spiccato period in a crisp, rapid spiccato (Guettler and Askenfelt 1998). During this contact time (between 40 and 60 ms depending on the spiccato rate), the force resembles a half period of a sine wave reaching peak values of typically 1.5 - 2 N (see Figure 4). The force waveform is somewhat peaked due to a "ripple" component at about 150 Hz. The magnitude of this ripple can change significantly for a slight change in driving frequency, depending on the relation to the spiccato rate (see Figure 5). For this bow, a shift in driving frequency of 1 Hz boosted the 150-Hz ripple markedly.

A modal analysis of two bows showed that the 150-Hz component can be traced down to two modes of the pivoted bow at approximately 150 Hz (#3) and 170 Hz (#4), see Figure 6. Mode #3 resembles the second mode of a free-free stick without hair (typically at 160 Hz, see Askenfelt 1992b), but with a marked rotational motion of the tip. The flexing of the stick in mode #4 occurs mainly in the outer part of the stick.

By driving the bow under the hair at the supporting point it was observed that modes #3 and #4 included a strong motion of the bow hair on one side of the support, in the longer part for the lower mode, and in the shorter for the higher. The Q-values of these modes are rather high, of the order of 30. The mode frequencies could be shifted by moving the support (spiccato point), and also brought to coincide in frequency. Rather than being a doublet with the hair and stick moving in-phase and out-of-phase, respectively, the observed vibration states are probably two different modes of the pivoted bow with a similar motion of the stick, but differing in the motion of the hair on each side of the supporting point. (cf. the free-free modes reported by Bissinger 1995). The question cannot be setttled completely until measurements have been taken also on the bow hair.

The 150-Hz ripple seen in the contact force is probably a combination of the two modes. The boost of the ripple in Figure 5 (bottom) was observed when the 13th component of the driving force coincided with one of the mode frequencies. Interestingly, the bouncing motion of the bow can be started by supporting the bow hair on a shaker at the spiccato point and driving with the frequency of mode #3 or #4. This indicates a non-linear coupling between these modes and the bounce mode, which possibly has something to do with a certain quality of some bows "to cling to the

Figure 6 ▪ Modal analysis of a violin bow showing mode #1, "bounce mode" (14 Hz), mode #2 (59 Hz), mode #3 (157 Hz), and mode #4 (169 Hz) for a case with boundary conditions corresponding to the string-contact part in spiccato. The bow is pivoting around an axis at the frog and the bow hair is fastened to a fixed support at a typical position for rapid spiccato 2 cm outside the midpoint. No measurements were taken on the bow hair. As a consequence the hair forms straight lines between the frog-support-tip for all modes.



string" during long notes in legato and detaché.

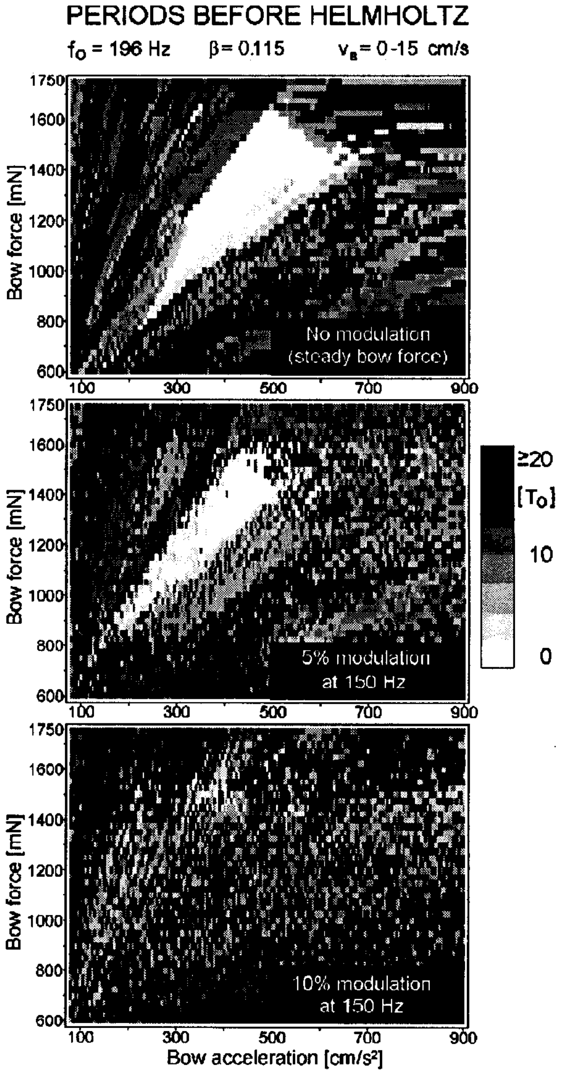
At the impact at landing, mode #2 of the pivoted bow just below 60 Hz will be excited (see Figure 6). This mode shows a large motion of the stick at the middle and a node at about 1/6 from the tip. This gives a pronounced motion of the outer part of the stick, which moves upwards and stretches the hair while the middle part moves downwards. The Q-value of this mode can be rather high, well above 50. However, no significant modulation of the bow force during the contact with the transducer (string) due to this mode was observed. Nevertheless, the possibility of a periodic variation in the hair tension and instantaneous velocity of the hair at the contact point remains.

The shape of 60-Hz mode of the pivoted bow in Figure 6 is similar to the lowest mode of the bow measured under free-free conditions and occurs at approximately the same frequency (cf. Askenfelt 1992b; Bissinger 1995). In line with these findings, a vibration at about 60 Hz was observed in the stick not only during string contact but also during off-string flight.

A Crisp Spiccato

A good bow will facilitate a rapid spiccato with clean, crisp attacks. In order to establish a prompt Helmholtz motion it seems essential that the bow force builds up fast to give the first slip after shortest possible delay. Further, the bow force should stay essentially constant during the initial periods in order to avoid premature (multiple) slipping or prolongation of periods. During the initial periods, the product of the triggering velocity pulse and the point impedance of the string is only a few percent higher than the limiting static frictional force of the preceding sticking interval (see Guettler 1992, Figure 3). Therefore, if the bow force decreases too fast after the first slip, the build-up in frictional force will trigger a release in the nominal stick interval, prior to return of the triggering pulse generated by the first slip. Also, if the bow force increases too rapidly, the triggering-pulse

Figure 7 ■ Effect of a modulation in bow force on the duration of the pre-Helmholtz transient (the duration of the aperiodic transient before periodic Helmholtz triggering has been established). Each panel shows 7755 simulations (pixels) with different combinations of bow force and bow acceleration.



Upper panel: Constant bow force; *middle:* 5 % amplitude modulation in bow force at 150 Hz; *lower panel:* 10% amplitude modulation in bow force at 150 Hz.

The light triangle-shaped areas indicate preferred parameter combinations giving short durations of the pre-Helmholtz transient. A constant bow force gives a large continuous area with many possible combinations of bow force and acceleration. The preferred area decreases considerably for 5% modulation and vanishes completely at 10% modulation, giving the player small chances of controlling the outcome of the attacks. The durations are given in nominal periods [T_0]. The bow is accelerating linearly from rest to a final velocity of 15 cm/s and the bow-bridge distance β is 1/8.7 = 0.115.

product will fail to reach the value of the limiting force after one period and chaotic behavior is likely to occur. Computer simulations show unambiguously that an unstable bow force during the note onset reduces the chances of reaching Helmholtz triggering quickly (see Figure 7). A box-shaped bow force with a steep slope at the onset which changes to constant value as soon as a sufficiently high force has been reached, would seem optimal for a crisp spiccato.

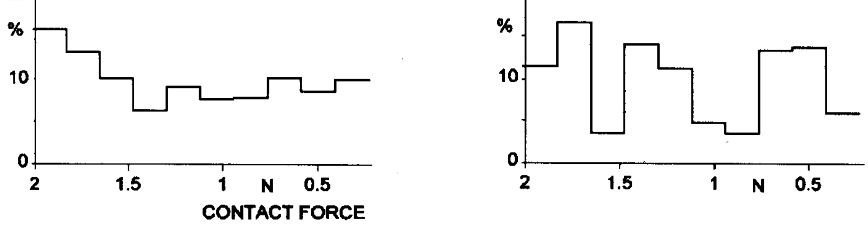
As seen in Figure 5, the 150-Hz component can steepen the slope at force build-up (from 0.12 to 0.23 N/ms), but on the other hand the accompanying stronger fluctuations in bow force give increased risk of multiple slipping. A histogram representation of the force histories in Figure 5 is given in Figure 8. A distribution with few values at intermediate force values and a concentration towards the maximum force percentile could be assumed to be advantageous according to the reasoning above. In this respect, the distribution for 12 Hz driving frequency with only minor 150-Hz activity would be more promising for a crisp spiccato. The damping due to the player's holding of the bow will reduce the 150-Hz ripple slightly, but in no way cancel this activity in the bow. The bow hold in spiccato is light, and located close to a nodal point for most bow modes (Askenfelt 1992b).

Comparisons between wooden bows of different quality and bows of novel materials (fiber glass, carbon fiber composites) gives a picture which is far from clear-cut. The strength of the 150-Hz ripple in spiccato seems not to be related to the rated quality of a bow in a simple way. It is present in poor bows as well as in excellent ones, the actual strength depending on the spiccato rate. Also a bow which had been bent to a convex ("baroque") camber which made it impossible to the tighten the hair to normal tension (25 N instead of 55 - 60 N) showed the ripple activity (now at 110 Hz) in the contact force. This bow performed very poorly in spiccato, probably depending on a low bounce mode frequency (10 Hz instead of 13 -15 Hz) due to the low tension of the hair. As noted by players, the ability to take a high tension of the bow hair with only a minor straightening of the camber is one of the basic quality marks of a good bow.

Conclusions

The rapid spiccato bowing is dependent on a bounce mode of the pivoted bow at about 13 - 15 Hz. This mode seems to be very similar for all bows. The contact force with the support showed a "ripple" component at about 150 Hz, which was traced to a combination of two modes with strong activity in the outer part of the bow. The strength of this ripple, which is dependent on the spiccato rate, can reduce the force buildup time. A short buildup time would facilitate a fast and crisp attack, but the accompanying variations in bow force may, on the other hand, increase the risk of multiple slipping. The net effect of the ripple is not known yet, but it seems plausible that it reduces the chances of a clean attack.

Figure 8 ▪ Histogram distributions of the contact force histories in Fig. 5 with little ripple at a spiccato rate of 12 Hz (*top*), and pronounced ripple at 13 Hz (*bottom*). The force increases in the negative direction (to the left) in conformity with the orientation of the force in Fig. 5 (downwards).



In view of that bows perform very differently in rapid spiccato, it seems reasonable that the differences must be sought in the action of the stick, and not in the basic bounce mode which can described by a lumped mass-spring system. The differences in masses and moments of inertia between bows are small and the hair is taut to nearly the same tension. In view of these similarities the 150-Hz ripple, and possibly also higher modes, are interesting properties to compare in a future study. ■**CASJ**

ACKNOWLEDGMENTS

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REFERENCES

Askenfelt, A. 1992a. "Properties of violin bows," *Proc. of International Symposium on Musical Acoustics, Tokyo, August 28 - September 1, 1992 (ISMA '92)*: 27-30.

Askenfelt, A. 1992b. "Observations on the dynamic properties of violin bows," *Speech Transmission Laboratory Quarterly Progress and Status Report*, Dept. of Speech Communication and Music Acoustics, Royal Institute of Technology, Stockholm, STL-QPSR 4/1992: 43-49.

Bissinger, G. 1995. "Bounce tests, modal analysis, and the playing qualities of the violin bow," *CAS Journal*, 2(8) (Series II): 17-22.

Guettler, K. 1992. "The bowed string computer simulated—some characteristic features of the attack," *CAS Journal*, Vol. 2, No. 2 (Series II): 22-26. Note the printing errors in the expressions preceding Eq. 3.2 which should read: $t_1 = 2X/C_{TRV}$ and $t_2 = 2(L-X)/C_{TRV}$

Guettler, K. and Askenfelt, A. 1998. "On the kinematics of spiccato and ricochet bowing" *CAS Journal* (November): 9-15.

Pitteroff, R. 1995. *Contact mechanics of the bowed string*, Ph.D. thesis, University of Cambridge, 1995.