RELATION BETWEEN BOW RESONANCES AND THE SPECTRUM OF A BOWED STRING

Knut Guettler

The Norwegian State Academy of Music
P.Box 5190 Majorstua,
N-0302 Oslo, Norway

Anders Askenfelt

The Royal Institute of Technology,
Dept. of Speech Communication & Music Acoustics, P.B. 70014,
S-10044 Stockholm, Sweden

ABSTRACT

The dynamics of the frictional force between the bow hair and the string forces the bow to oscillate in its longitudinal direction during the stroke. Under certain conditions, such bow oscillations can be observed in the force spectrum of the string, and also in the velocity spectrum of the bridge. The present work discusses the conditions necessary for such an interaction to be noticeable. Furthermore, the influence of bridge resonances on the spectrum of the frictional force is examined. This analysis is based on computer simulations in combination with observations of real bows played by a bowing machine.

1. INTRODUCTION AND OVERVIEW

Every string player "*knows*" that "*different bows produce different sounds on the instrument*". Also, "*some bows are easier to play than others*". The explanation to such statements could be searched for in the small fluctuations of velocity which the bow hair makes with respect to the inert "bow velocity" performed by the player. As this paper will show, these fluctuations indeed exist, but there is no simple relation between their magnitudes and their impact on the string force spectrum at the bridge: the transfer function is composed of several elements, some of which shall be examined here.

1.1 Bow resonances

Bow resonances have been reported by Schumacher [1] and others. Such resonances are initiated by changes in the frictional force, which are present during all of the fundamental period, but most significantly during the static-friction part of it, and the transitions between

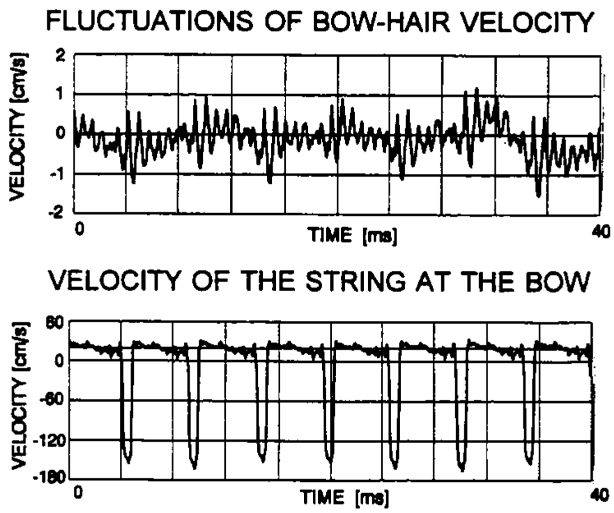

Figure 1: During a steady-state stroke with velocity 20cm/s and bow force 800mN, the bow hair fluctuates with amplitudes up to 1.5 cm/s. Frequencies lower than the string's fundamental are evidently present.

frictional forms (slip/stick). The bow resonances will couple to the string resonances and may transmit, reflect and/or absorb some part of the arriving energy, depending on their admittance ratios.

1.2 Bow and string admittances

During the stick (static-friction) interval, the frictional force works on three admittances in mechanical series: the frequency-dependent admittance of the bow, and the transverse and torsional admittances on a string with reflecting terminations. The ratios of these vary grossly with frequency. A simple equation for point admittance of a string with reflecting terminations can be expressed as shown below, provided the reflection functions adjust for all losses during propagation in the string (Guettler [2]):

$$Y(x, j\omega) = \frac{1}{2Z}\left[\frac{1}{1+R_{BR}(j\omega)\ e^{-j\omega 2X/C}} + \right.$$

$$\left. + \frac{1}{1+R_{NUT}(j\omega)\ e^{-j\omega 2(L-X)/C}} - 1\right]^{-1} \qquad (1)$$

where:

$Y(x, j\omega)$ = *string point admittance.*
Z = *characteristic wave resistance of the string.*
x = *distance from bridge to point of excitation.*
L = *length of the string.*
ω = *angular frequency* ($2\pi f$).
C = *wave velocity.*
$R_{BR}(j\omega)$ = *reflection at the bridge.*
$R_{NUT}(j\omega)$ = *reflection at the nut.*

When comparing a transverse system of high Q-value modes to a low-Q-value torsional system, the point-admittance curve of the latter converges much faster (toward half the characteristic wave admittance) in the high-frequency end. This implies greater torsional influence on the string surface admittance ($Y_{TRV} + Y_{TOR}$) at the low frequency harmonics.

1.3 Bridge/string transfer function.

The function of velocity transference from one arbitrary point on the string to the bridge can (respecting the conditions of Eq.1) be expressed by (Guettler [2]):

$$\frac{v_{BR}(j\omega)}{v_X(j\omega)} = \frac{[R_{BR}(j\omega)+1]e^{-j\omega X/C}}{1+R_{BR}(j\omega)e^{-j\omega 2X/C}} \qquad (2)$$

where:

$v_{BR}(j\omega)$ = *velocity of the bridge.*

Besides being related to the bridge admittance, this function is sinusoidal, and favours incomparably frequencies close to $f_0 n/\beta$, which on the other hand give low admittances. ($n = 1,2,3...$, $f_0 = 1.$ mode frequency, $\beta = x/L$.) If the relatively modest resonant fluctuations of bow velocity shall have any impact on the bridge-velocity spectrum, chances are indeed greatest near to $f_0 n/\beta$.

1.4 Spectrum of the frictional force

During the sticking interval, a steady-state bow velocity forces the string to follow in the same direction. However, due to string end reflections, the frictional force will be varying during this interval. Most important are the periodic reflections of transverse waves travelling "ping -pong" between the bow and the bridge. These waves are (mainly) excited at the release and capture of the string at the bow (Schumacher [3]), and rotate with a frequency equal (or close) to f_0/β. For this frequency, the transverse point impedance shows a peak, making the string harder to excite. Hence, a significant peak will also occur in the frictional-force spectrum. *The height of this peak is, however, reduced if the torsional impedance - the bridge impedance - or the bow-hair impedance shows a valley in the same region.* Simulations show that the most dominant maxima of the frictional-force spectrum are usually found at f_0, and near to f_0/β. When considering that bow-velocity fluctuations in principle may be regarded as a convolution between the impulse response at the bow hair, and the derivative of the frictional force, the importance of the friction spectrum with respect to bow resonances, becomes obvious.

1.5 Spectral magnitudes of string velocity at the point of bowing

Compared to our measured spectral amplitudes of the bow hair, the amplitudes of the string at the bowing point are generally very high. Per example: with normal values of β, the amplitude of f_0 will be close to $2V_{BOW}$, i.e., twice the steady-state bow velocity. This, of course, reduces the possibilities of timbral impacts caused by bow-velocity fluctuations. However, around the node frequencies, the sinusoidal string-velocity spectrum have notches. By the same frequencies, the frictional-force spectrum shows *peaks*, so chances are that impact of bow resonances on the string may be competitive just in these regions.

1.6 Time windows of static- and sliding friction

During the sliding interval, the string is more or less "decoupled" from the bow hair. Hence, the transmission of bow resonances takes much lower values. But, during the static interval, waves have been emitted on the string, equally to the bridge and the nut side of the bow, therefore, during the sliding interval, waves will arrive at (and mostly pass) the bow after having been reflected at the nut at a time $0.5(1-\beta)/f_0$ earlier. The frequency content of these waves will then vary in phase compared to the phases of the "decoupled" bow oscillations. Dependent on their relative phase angles (and thus β), this time window will very much determine the effect of decoupling on the string spectrum, when averaged over the full period: frequencies near $nf_0 = f_0(m + 0.5)/\beta$ give the strongest bow/string transference (n and m being integers: $0 \leq m < n$), while frequencies near $nf_0 = m/\beta$, $(0 < m < n)$ transfer up to about 2β times less. This rule of thumb holds only if we reckon a 180 degree phase shift at the nut reflection and ignore the differences of string admittances, as expressed through Eq.(1). The time windows may, due to the on/off switching effect, also produce "sidelobe frequencies", although most probably of insignificant magnitudes, compared to those already in the string.

1.7 Other elements that may cause audible changes of timbre

(1) The violin bow has major resonances below the frequency range of the instrument - at frequencies where the violin body is a poor radiator. There is nonetheless a possibility that these low frequencies are perceived by the ear through amplitudal modulation of the "real" violin frequencies. The ear cannot always easily judge which is which. At any rate, during the attack transients, the frictional force fluctuates vividly, also at lower frequencies than the fundamental f_0.
(2) As described by McIntyre et al. [4], the length of fundamental periods tends to fluctuate somewhat. Simulations show that both string torsion and bow resonances could cause such fluctuations, as both may possess mode frequencies interfering with those in the transverse plane. Apart from the "phaser effect" of such behaviour, a widening of the peaks in the power spectrum may increase the power of (resonant) "near-harmonic" frequencies, the same way a vibrato does.

2. COMPARISON OF STRING AND BOW ADMITTANCES

2.1. Transverse and torsional point admittances

Figure2 shows transverse and torsional point admittances of a "heavy" violin G string excited at $\beta = 0.08$. These values were acquired through computer modeling, by averaging FFTs of a series of "white-noise" simulations on a string with transverse and torsional wave impedances of 370 and 925 g/s, respectively. (Pickering [5] gives values from 274 to 386 g/s for transverse wave impedances of violin G strings, while the impedances for higher strings, are generally lower.) The relative velocity C_{TOR}/C_{TRV} was programmed to 4.8, and Q-values to $245<Q_{TRV}<525$ and $17<Q_{TOR}<31$ within the 10,000 Hz bandwidth. The simulation parameters above are used throughout this text, including the figures.

As can be read from Fig. 2, the transverse admittance is by far the highest for most mode frequencies. This implies that little transverse kinetic energy will transform into torsional at these harmonics. However, at the 11th and 12th harmonic (near f_0/β), this difference is small. Had in our example, the relative velocity been equal to 4.0, a substantial amount of the transverse kinetic energy would transform into torsional at these frequencies, because the torsional admittance would have its third-mode *peak* at $12 f_0$. On the other hand, the 9th and 10th would benefit from this change, then being in the range of a torsional admittance valley. When comparing simulations applying these two relative velocities, a very noticeable difference (6 - 8 dB or more) in the

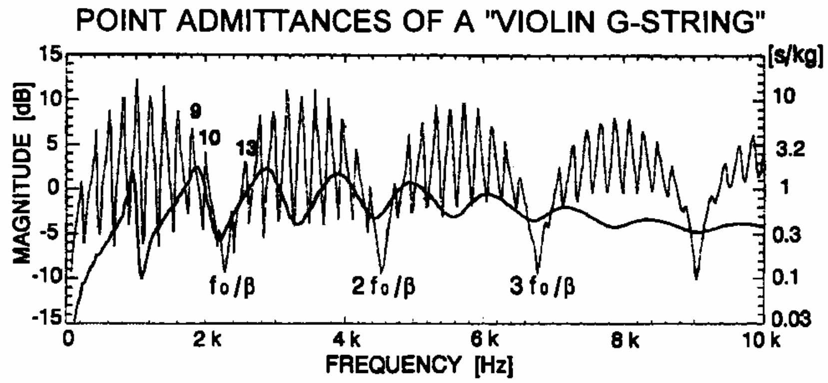

Figure 2: Transverse and torsional point admittances for a high tension violin G string (curves obtained through computer simulations: for details on string properties see text). At most mode frequencies, the transverse admittance is significantly higher than the torsional.

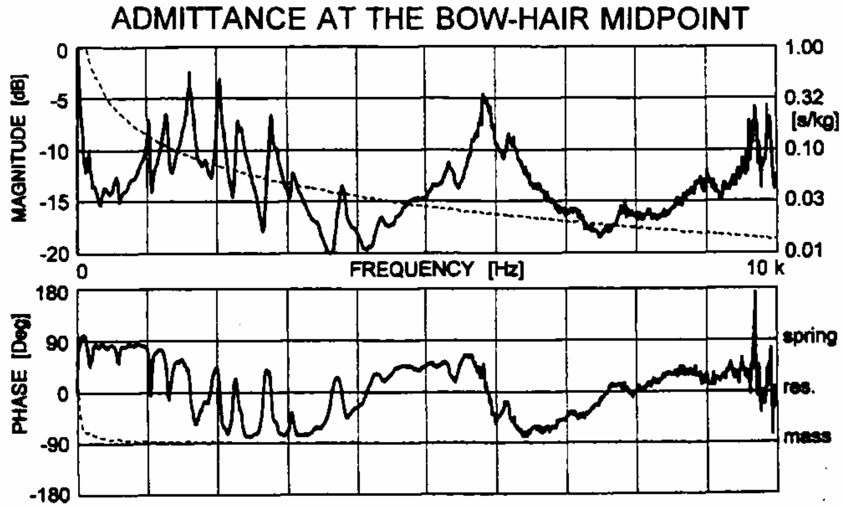

Figure 3: Admittance of a bow, measured with an accelerometer in the bow hair. The impedance of the accelerometer is accounted for, and drawn as dashed lines. The frequency region above 5kHz should be considered with caution, as it may contain spuriðus information.

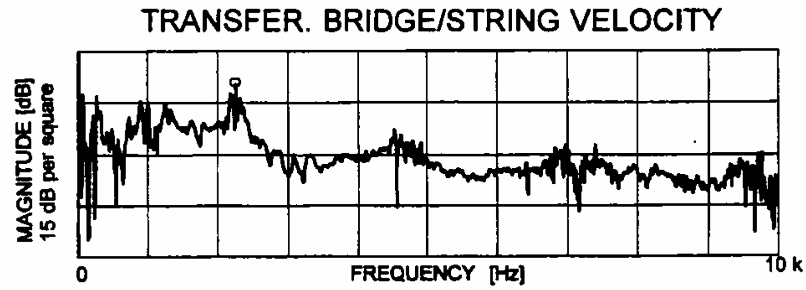

Figure 4: Transfer function: bridge velocity divided by velocity of the string at the bowing point. The sinusoidal (phase-related) bow-to-bridge signal transference is quite visible, superimposed on the bridge transmission curve. The frequency $C_{TRV}/2X \approx 2.25$ kHz is circled.

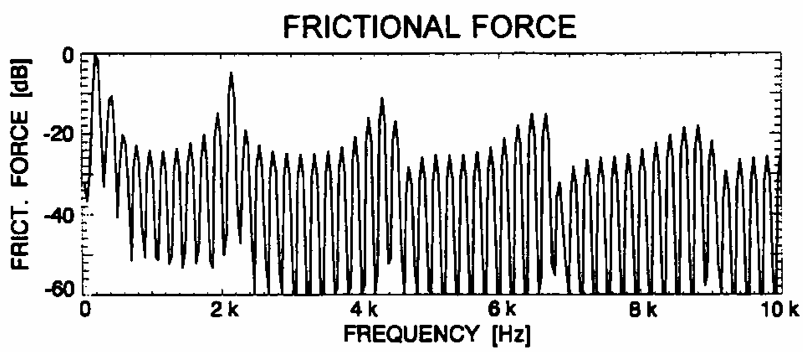

Figure 5: Simulated frictional-force spectrum of an open G string bowed at $\beta = 1/11.5$ (as for Figures 2 and 4). High amplitudes occur at f_0 and at integer multiples of f_0/β.

bridge power spectrum can be observed for some of these frequencies.

Schumacher [3] has reported that formally, the bow admittance can be viewed in the same way as one views the rotational string modes. Consequently, the bow possesses the same potentials of reaction, provided its admittance be high enough compared to the string-point admittance.

2.2 Admittance of the bow (hair) in its longitudinal direction

In the present experiment, a miniature accelerometer was fastened in the bundle of hair (with all hairs in contact), and hit with a miniature force hammer. Measurements were taken both in the longitudinal and transverse direction of the bow hair, and at different positions along the bow (tip - middle - frog). The bow was mounted in the flexible bow holder of the bowing machine, with the hair free from contact with the strings. (With the flexible bow holder, the bow resonances are more closely matching those obtained when holding the bow by hand.)

Fig. 3 shows the admittance obtained when measuring the longitudinal admittance of the bow hairs at their midpoint. The measurement is influenced by, the accelerometer, the mass of which (1.1 g) is of the same magnitude as that of the bow hair (a complete bundle of bow hairs weighs between 4 and 5 g). A compensation is therefore necessary, and has been calculated for. As the compensation is relatively large, even a small uncertainty in the compensation admittance will have a rather large influence on the result, and in particular the phase information.. We should therefore be somewhat cautious when considering the measurements above, say 5 kHz. With respect to individual bow characteristics, the major wood resonances fall below this frequency anyhow.

Different string positions on the bow hair produce different resonances, particularly above 2 kHz. Some of these have been seen to surpass the values of Fig. 3. In general however, the admittance of the bow as "seen by the string", is much lower than are the string admittances for the transverse string-mode frequencies. (Further discussion on the origins of bow resonances is beyond the scope of this article.) For higher tuned strings, possessing (up to some 3 - 4 dB) higher admittances, the admittance gap is even greater, and chances of spectral influence are reduced accordingly. On other bowed instruments of lower frequencies, the admittances may be "better matched". Which bow being the superior - the resonant or the non-resonant - remains to be evaluated, however.

3. THE TRANSFER FUNCTION

Figur 4 shows the measured transfer function of a violin G string: velocity of the bridge divided by string velocity at the bow. The bowing point is equal to L/11.5 as in Fig. 2. The sinusoidity is clearly visible. Compared to Eq.(2), the measured function bears typical features of string stiffness (expanding intervals between peaks toward the high-freqency end) and energy dissipation (lowered transfer ratio for high frequencies).

The transfer function was measured in a rather unsophisticated way, just for demonstration of the sinusoidity: with the bow held by the bowing machine (thus maintaining a correct bowing point) it was manually "scratched" over the string, exciting only a thin hiss. The string, at its nut side, was efficiently dampened by pads of foam rubber. Since the distance between the string and the magnet below would vary, no calibration was possible. Fig. 4 shows the average of 10 such registrations.

4. THE FRICTIONAL FORCE

Figure 5 shows the (simulated) frictional force of the string model bowed with fixed velocity and (high) bow force. The frequencies f_0 and f_0/β dominate. Besides what was already discussed under paragraph 1.4, the frictional-force spectrum is affected by the bridge impedance to the extent the string point admittance is affected through the bridge reflection function.

At frequencies near $nf_0/\beta \approx nC_{TRV}/(2L\beta)$ (n = pos. integer), where $\cos(\omega 2X/C_{TRV}) = 1$, the expression $1+R_{BR}(j\omega)$ of Eq.1 can be substituted with expressions of impedance: $1+R_{BR}(j\omega) = 2Z/[Z+Z_{BR}(j\omega)]$, where $Z_{BR}(j\omega)$ is the impedance of the unstrung bridge. The effect of bridge impedance on the transverse string point admittance can then more easily be seen: *the string point admittance increases with increasing bridge admittance.* Between these "node" frequencies, the influence of the bridge impedance is reduced and varies sinusoidally.

5. CONCLUSIONS

Admittances of the string at the bowing point, and of the bow at its hair, have been calculated and measured, respectively. In certain frequency regions, partly dependent on the bow position, the transverse point admittance of the string is found to have magnitudes small enough to be comparable to the peak admittances of the bow. Even though significant fluctuations of the steady-state bow velocity were observed in the bow hair, no hard evidence was found of these reaching magnitudes that substantially would influence the output spectrum, as yet.

This study was mainly directed toward steady-state behaviour of the bow, investigating frequencies within the range of natural string harmonics. Low-frequency oscillations during transients should be subject to investigation next.

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