Integration is a mathematical technique for converting from one dimension to another one, e.g., from *acceleration* to *velocity*:

Example: while *acceleration* can be expressed with the dimension meter per second squared [m/s^2], *velocity* would have the dimension meter per second [m/s]. In order to convert from acceleration to velocity, the acceleration has to be *integrated* with respect to time (t) with dimension second [s]. This is written:

$$Velocity + C = \int Acceleration\ d(t),$$

where t stands for *time*, $d(\cdot)$ implies "as derivative of", and C is a *constant term* (also with dimension [m/s]).

Notice: integration always implies the inclusion of a constant term, which defines the boundary condition (here it is the starting velocity before acceleration takes place).

In the present case a further integration (of Velocity + C) with respect to time would have given *position* (including a constant starting position) as the result, with the dimension meter, [m].

The inverse of integration is **differentiation**. After a function is differentiated you end up with its **derivative**.

E.g., non-zero *acceleration* is the derivative of *velocity* changing as function of time, while non-zero *velocity* is the derivative of *position* changing with respect to ditto. The integral of zero acceleration is in principle a constant velocity, due to the constant term, etc.

Notice: because *derivation* removes any initial boundary constant, the corresponding *integral* is *not* uniquely defined unless that constant is known. On the other hand, the *derivative* remains the same and is uniquely defined, regardless of any initial constant.

Graphic example:

In Fig. 1 below, example plots of *acceleration*, *velocity*, and *position* are shown for a given case. If we start with the middle panel, *velocity*, we see that the slope of the curve (exemplified by red lines, tangential to the velocity curve—with dimension m/s per second) for the most part varies with time. Its derivative, *acceleration*, is no more than a description of this variation. To go from acceleration to velocity (i.e., to *integrate* the acceleration) we may imagine how the area (marked yellow) below the acceleration curve between time zero and an increasing t changes as t runs from 0 to 6 seconds.

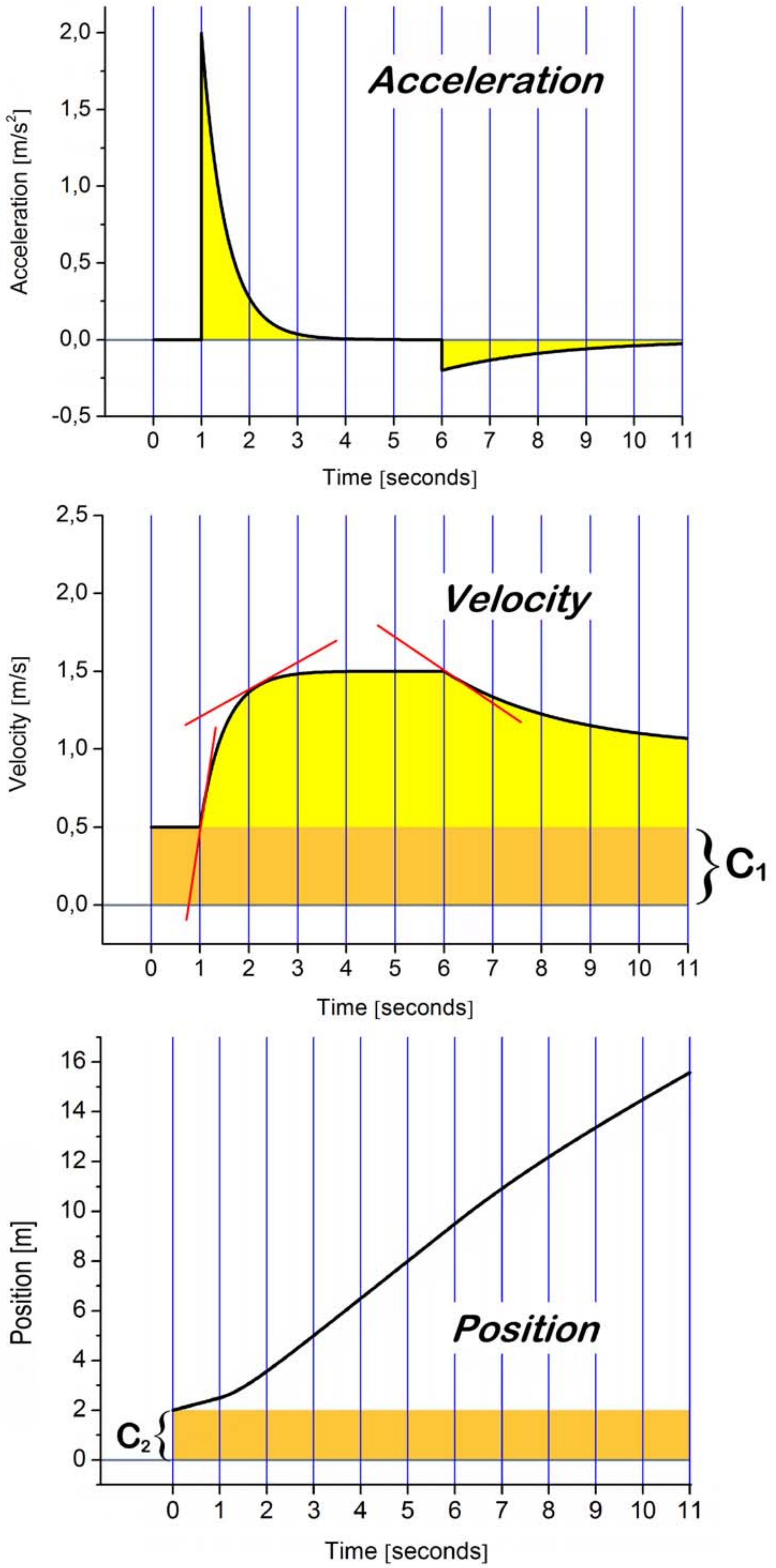

Figure 1: See text

The velocity curve describes this development of area change *plus a constant starting velocity*, C_1 of 0.5 meter per second (marked darker yellow), which was invisible in the acceleration plot. Between *t* = 6 and 11 seconds, a negative acceleration takes place, and the area between the acceleration curve and the zero line now has to be *deducted* from the acquired velocity present a time *t* = 6. We thus notice that from this point of time on, the velocity is decreasing in the middle plot even though the corresponding acceleration is increasing.

To calculate how far the moving object did reach during these eleven seconds, we have to integrate once more, which in mathematical semantics may be written:

$$position = \iint_{t=0}^{11} acceleration\; d(t)d(t).$$

Once again we calculate the accumulated area under the curve, this time the velocity curve, to obtain *position*. And, since the given initial position was 2 meters rather than zero, we have to add another constant term C_2 = 2 m to the accumulated area change. We experience that during the eleven seconds, the object moved from position 2 m to position 15.6 m, a stretch of 13.6 meters.