

**Impedance** (in this connection “mechanical impedance”, commonly used symbol:  $z$ ) describes an object’s resistance to being moved. The resistance can in principle take a number of forms:

Resistance (or “viscous resistance”) [ $\text{kg/s}$  = kilograms per second]: works against *velocity* (like a dashpot), implying that the Force (expressed in newtons [N], i.e., meters  $\times$  kilograms / seconds squared) required to move the object, increases linearly with the obtained speed of the object moved: Force [ $\text{m kg/s}^2$ ]/Resistance [ $\text{kg/s}$ ] = Velocity [ $\text{m/s}$ ], i.e., meter per second.

Stiffness [ $\text{kg/s}^2$ ] works against *the displacement of an object* from its equilibrium. E.g., moving an object with stiffness  $1 \text{ kg/s}^2$  one meter, requires a force of one Newton: Force [ $\text{m kg/s}^2$ ]/Stiffness [ $\text{kg/s}^2$ ] = Position [ $\text{m}$ ].

Mass [ $\text{kg}$ ] works against acceleration [ $\text{m/s}^2$ ]. (Once the mass has accelerated up to a certain velocity, it will continue moving with that velocity if the force is suddenly removed and no other forces are acting.) Force [ $\text{m kg/s}^2$ ]/Mass [ $\text{kg}$ ] = Acceleration [ $\text{m/s}^2$ ].

Resistance, Stiffness, and Mass may well all be present in one single object. Impedance comprises the sum of Mass + Resistance + Stiffness, expressed in a complex number with a real part and an imaginary part. If we have the combination of Mass and Stiffness (without Resistance) set in motion, it will oscillate forever with the Frequency [ $\text{Hz} = 1/\text{s}$ ]:

$$Frequency = \frac{1}{2\pi} \sqrt{\frac{Stiffness}{Mass}}$$

If we also include the Resistance, the oscillations will fade out with a certain (logarithmic) pace, depending on its value: the higher the resistance, the faster the decay.

To picture such an object we can (although physically not quite correctly) imagine a rubber ball that with its mass, resistance, and stiffness (springiness).

Impedance is often plotted as Magnitude and Phase plots in the frequency domain, where pure Resistance is referred to as  $0^\circ$  (equal to 0 radians), while pure Mass and Stiffness (most often) are indicated as  $+90^\circ$  and  $-90^\circ$  (equal to  $+\pi/2$  and  $-\pi/2$  radians), respectively.

The inverse of impedance is “Admittance”, also called “Mobility”.

(See also for strings: “Characteristic wave resistance”, or “Characteristic wave impedance”).)

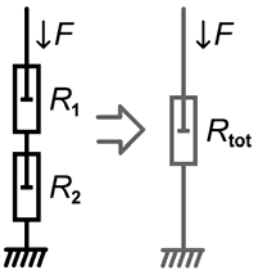
### Calculation with impedances:

Quite often you have the force acting on more than one impedance. In the following we shall give examples of how a combination of two impedances can be recalculated to give the total impedance. Even though this kind of calculation works also for complex impedances, we shall for the sake of simplicity be using two impedances consisting of resistances only as our examples:



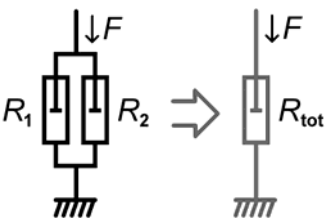
The graphic symbols here indicate:  $\downarrow F$  = force (with direction) [N];  $R$  = Resistance (“dashpot”) [kg/s]; the lower symbol (the “comb”) marks a fixed point.

If you have two mechanical impedances, here dashpots (resistances) in series, with values  $R_1$  and  $R_2$ , respectively (see the figure below), the resulting resistance will be  $R_{tot} = R_1 \times R_2 / (R_1 + R_2)$ .



For those of you familiar with electric circuits, you will notice that this calculation equals the calculation of electrical resistances in parallel. Given the values  $R_1 = 1$  kg/s and  $R_2 = 2$  kg/s,  $R_{tot}$  becomes  $2/3$  kg/s.

If you have two mechanical impedances, here dashpots (resistances) in parallel, with values  $R_1$  and  $R_2$ , respectively (see the figure below), the resulting resistance will be their sum:  $R_{tot} = R_1 + R_2$ .



For those of you familiar with electric circuits, you will notice that this calculation equals the calculation of electrical resistances in series. Given the values  $R_1 = 1$  kg/s and  $R_2 = 2$  kg/s,  $R_{tot}$  becomes  $3$  kg/s.

When calculating with mobilities (admittances),  $y$ , instead of impedances, the mobilities of a mechanical series can be calculated as their sum, while for mechanical parallel, they should be calculated as

$$\frac{1}{\frac{1}{y_1} + \frac{1}{y_2} + \dots + \frac{1}{y_n}}$$

On the string surface, the characteristic wave resistances for transverse and torsional waves are acting in series, i.e., yielding together more than one alone.