Convolution (often indicated with an asterisks *; its mathematical expression sometimes referred to as a *folding integral*) is a mathematical operation for calculating a joint function from two individual functions, e.g., an impulse-response function and a force function. The convolved function may be *continuous* (as a result of an integral) or discrete (as function of sums of digitalized, sampled values). The easiest way to explain the technique is to look at an example of the latter:

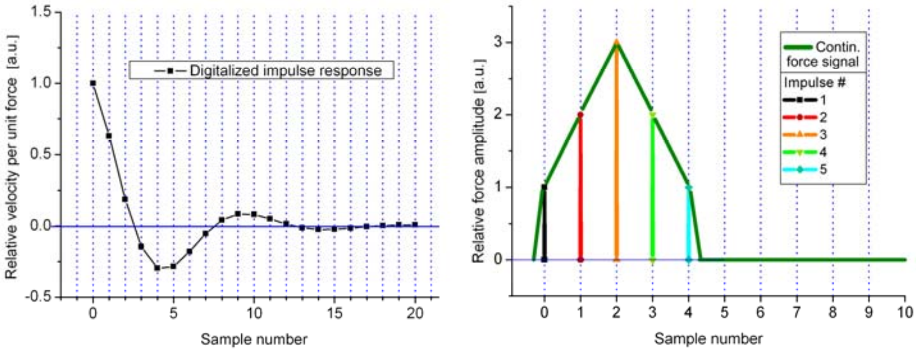

Figure 1: Impulse response (left panel) to be convolved with a force function (right panel). See text. Notice: "a.u." stands for *arbitrary units*.

In Fig. 1, an impulse response is shown at the left panel. We want to know how this system will behave when excited by the force function of the right panel (olive line). When we digitalize the continuous force function shown here, we get the values 1, 2, 3, 2, 1, 0, 0, 0,… for the respective samples starting at sample number zero. If we consider a series of impulses given these values (black, red, orange, green, and cyan for the non-zero ones, respectively), we can consider a bundle of individual responses the way it is plotted in Fig. 2, left panel:

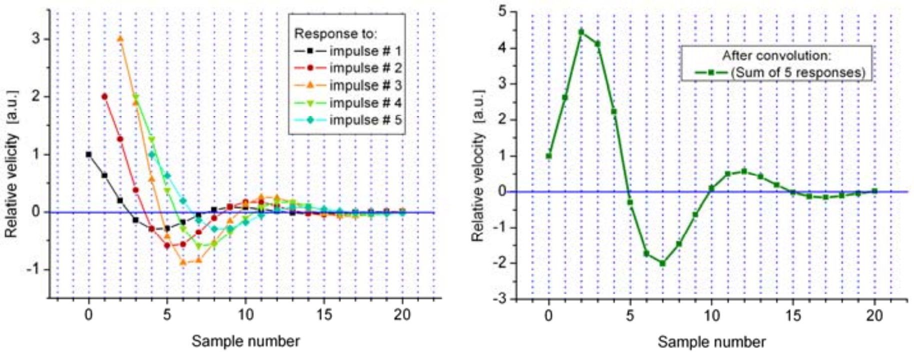

Figure 2: Left panel—Individual responses to impulses 1 through 5 of Fig. 1. Right panel—The resulting function after convolution between the impulse response and the force function.

Notice that the individual responses are shifted in time according their respective sample positions (sample numbers) of Fig. 1. When adding together the responses for each sample number (time step) we get the function of the right-hand panel of Fig. 2: This is the function of the system's impulse response and the force function convolved. I.e., this is how the system would have behaved if it was excited with a force function shaped like a tent, as shown in Fig. 1, right panel.

For a conversion from the discrete, digitized version of convolution to the continuous ditto, one may imagine the sampling period to approach zero, so that an increasing number of individual, quasi infinitesimally narrow impulses are positioned right next to each other, without any interval in between. However, in the case of the truly continuous version (different from the discrete one) both the impulse response and the force function have to be describable as mathematical functions.

In practice, discrete convolution is most often performed by multiplying the (complex) frequency spectra of the two functions, followed by an inverse transform back to the time domain. FFT (Fast Fourier Transform) is well suited for this operation. In such cases zeroes must be added to each signal array in the time domain before FFT and convolution, so that the number of elements becomes equal or greater than the sum of the elements in the original signals cropped.

Deconvolution is the opposite process of convolution. E.g., the function shown in the right panel of Fig.2 may be deconvolved by the force function shown in the right panel of Fig. 1 to yield the impulse-response function in the left panel of Fig. 1. This is what is done with bridge responses excited by force hammer impacts, in order to get the true impulse response. Practically, one can *divide* the measured response spectrum by the spectrum of the hammer impact and then do an inverse FFT, to get the impulse response. (In this case, no zero-padding is required.)